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XXIV. *On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of Life Contingencies. In a Letter to FRANCIS BAILY, Esq. F.R.S. &c. By BENJAMIN GOMPERTZ, Esq. F.R.S.*

Read June 16, 1825.

DEAR SIR,

THE frequent opportunities I have had of receiving pleasure from your writings and conversation, have induced me to prefer offering to the Royal Society through your medium, this Paper on Life Contingencies, which forms part of a continuation of my original paper on the same subject, published among the valuable papers of the Society, as by passing through your hands it may receive the advantage of your judgment.

I am, Dear Sir, yours with esteem,

9th June 1825.

BENJAMIN GOMPERTZ.

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## CHAPTER I.

ARTICLE 1. IN continuation of Art. 2. of my paper on the valuation of life contingencies, published in the Philosophical Transactions of this learned Society, in which I observed the near agreement with a geometrical series for a short period of time, which must pervade the series which expresses the number of living at ages in arithmetical progression, pro-

ceeding by small intervals of time, whatever the law of mortality may be, provided the intervals be not greater than certain limits: I now call the reader's attention to a law observable in the tables of mortality, for equal intervals of long periods; and adopting the notation of my former paper, considering  $L_x$  to express the number of living at the age  $x$ , and using  $\lambda$  for the characteristic of the common logarithm; that is, denoting by  $\lambda(L_x)$  the common logarithm of the number of persons living at the age of  $x$ , whatever  $x$  may be, I observe that if  $\lambda(L_n) - \lambda(L_{n+m})$ ,  $\lambda(L_{n+m}) - \lambda(L_{n+2m})$ ,  $\lambda(L_{n+2m}) - \lambda(L_{n+3m})$ , &c. be all the same; that is to say, if the differences of the logarithms of the living at the ages  $n, n+m, n+2m, n+2m, n+3m$ ; &c. be constant, then will the numbers of living corresponding to those ages form a geometrical progression; this being the fundamental principle of logarithms.

Art. 2. This law of geometrical progression pervades, in an approximate degree, large portions of different tables of mortality; during which portions the number of persons living at a series of ages in arithmetical progression, will be nearly in geometrical progression; thus, if we refer to the mortality of DEPARCIEUX, in Mr. BAILY's life annuities, we shall have the logarithm of the living at the ages 15, 25, 35, 45, and 55 respectively, 2,9285; 2,88874; 2,84136; 2,79379; 2.72099, for  $\lambda(L_{15})$ ;  $\lambda(L_{25})$ ;  $\lambda(L_{35})$ ; &c. and we find  $\lambda(L_{25}) - \lambda(L_{15}) = .04738$   $\lambda(L_{35}) - \lambda(L_{25}) = .04757$ , and consequently these being nearly equal (and considering that for small portions of time the geometrical progression takes place very nearly) we observe that in those tables the numbers of

living in each yearly increase of age are from 25 to 45 nearly, in geometrical progression. If we refer to Mr. MILNE's table of Carlisle, we shall find that according to that table of mortality, the number of living at each successive year, from 92 up to 99, forms very nearly a geometrical progression, whose common ratio is  $\frac{3}{4}$ ; thus setting out with 75 for the number of living at 92, and diminishing continually by  $\frac{1}{4}$ , we have to the nearest integer 75, 56, 42, 32, 24, 18, 13, 10, for the living at the respective ages 92, 93, 94, 95, 96, 97, 98, 99, which in no part differs from the table by  $\frac{1}{37}$ th part of the living at 92.

Art. 3. The near approximation in old age, according to some tables of mortality, leads to an observation, that if the law of mortality were accurately such that after a certain age the number of living corresponding to ages increasing in arithmetical progression, decreased in geometrical progression, it would follow that life annuities, for all ages beyond that period, were of equal value; for if the ratio of the number of persons living from one year to the other be constantly the same, the chance of a person at any proposed age living to a given number of years would be the same, whatever that age might be; and therefore the present worth of all the payments would be independent of the age, if the annuity were for the whole life; but according to the mode of calculating tables from a limited number of persons at the commencement of the term, and only retaining integer numbers, a limit is necessarily placed to the tabular, or indicative possibility of life; and the consequence may be, that the value of life annuities for old age, especially where they are

deferred, should be deemed incorrect, though indeed for immediate annuities, where the probability of death is very great, the limit of the table would not be of so much consequence, for the present value of the first payment would be nearly the value of the annuity.

Such a law of mortality would indeed make it appear that there was no positive limit to a person's age; but it would be easy, even in the case of the hypothesis, to show that a very limited age might be assumed to which it would be extremely improbable that any one should have been known to attain.

For if the mortality were, from the age of 92, such that  $\frac{1}{4}$  of the persons living at the commencement of each year were to die during that year, which I have observed is nearly the mortality given in the Carlisle tables between the ages 92 and 99,\* it would be above one million to one that out of three millions of persons, whom history might name to have reached the age of 92, not one would have attained to the age of 192, notwithstanding the value of life annuities of all ages above 92 would be of the same value. And though the limit to the possible duration of life is a subject not likely ever to be determined, even should it exist, still it appears interesting to dwell on a consequence which would follow, should the mortality of old age be as above described. For, it would follow that the non-appearance on the page of history of a single circumstance of a person having arrived

\* If from the Northampton tables we take the numbers of living at the age of 88 to be 83, and diminish continually by  $\frac{1}{4}$  for the living, at each successive age, we should have at the ages 88, 89, 90, 91, 92, the number of living 83; 61.3; 45.9; 34.4; 25.8; almost the same as in the Northampton table.

at a certain limited age, would not be the least proof of a limit of the age of man ; and further, that neither profane history nor modern experience could contradict the possibility of the great age of the patriarchs of the scripture. And that if any argument can be adduced to prove the necessary termination of life, it does not appear likely that the materials for such can in strict logic be gathered from the relation of history, not even should we be enabled to prove (which is extremely likely to be the state of nature) that beyond a certain period the life of man is continually becoming worse.

Art. 4. It is possible that death may be the consequence of two generally co-existing causes ; the one, chance, without previous disposition to death or deterioration ; the other, a deterioration, or an increased inability to withstand destruction. If, for instance, there be a number of diseases to which the young and old were equally liable, and likewise which should be equally destructive whether the patient be young or old, it is evident that the deaths among the young and old by such diseases would be exactly in proportion of the number of young to the old ; provided those numbers were sufficiently great for chance to have its play ; and the intensity of mortality might then be said to be constant ; and were there no other diseases but such as those, life of all ages would be of equal value, and the number of living and dying from a certain number living at a given earlier age, would decrease in geometrical progression, as the age increased by equal intervals of time ; but if mankind be continually gaining seeds of indisposition, or in other words, an increased liability to death (which appears not to be an unlikely supposition with respect to a great part of life, though

the contrary appears to take place at certain periods) it would follow that the number of living out of a given number of persons at a given age, at equal successive increments of age, would decrease in a greater ratio than the geometrical progression, and then the chances against the knowledge of any one having arrived to certain defined terms of old age might increase in a much faster progression, notwithstanding there might still be no limit to the age of man.

Art. 5. If the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time, he lost equal portions of his remaining power to oppose destruction which he had at the commencement of those intervals, then at the age  $x$  his power to avoid death, or the intensity of his mortality might be denoted by  $aq^x$ ,  $a$  and  $q$  being constant quantities; and if  $L_x$  be the number of living at the age  $x$ , we shall have  $a L_x \times q^x \dot{x}$  for the fluxion of the number of deaths  $= -(L_x) \dot{}$ ;  $\therefore abq^x = - \frac{\dot{L}_x}{L_x}$ ,  $\therefore abq^x = - \text{hyp. log. of } b \times \text{hyp. log. of } L_x$ , and putting the common logarithm of  $\frac{1}{b} \times$  square of the hyperbolic logarithm of  $10 = c$ , we have  $c.q^x = \text{common logarithm of } \frac{L_x}{d}$ ;  $d$  being a constant quantity, and therefore  $\frac{L_x}{d}$  or the number of persons living at the age of  $x = d \cdot \bar{g}^{q^x}$ ;  $g$  being put for the number whose common logarithm is  $c$ . The reader should be aware that I mean  $\bar{g}^{q^x}$  to represent  $g$  raised to the power  $q^x$  and not  $g^q$  raised to the  $x$  power; which latter I should have expressed by  $\bar{g}^{q^x}$ , and which would evidently be equal to  $g^{qx}$ . I take this opportunity to make this observation, as algebraists are sometimes not sufficiently precise in their notation of exponentials.

This equation between the number of the living, and the age, becomes deserving of attention, not in consequence of its hypothetical deduction, which in fact is congruous with many natural effects, as for instance, the exhaustions of the receiver of an air pump by strokes repeated at equal intervals of time, but it is deserving of attention, because it appears corroborated during a long portion of life by experience; as I derive the same equation from various published tables of mortality during a long period of man's life, which experience therefore proves that the hypothesis approximates to the law of mortality during the same portion of life; and in fact the hypothesis itself was derived from an analysis of the experience here alluded to.

Art. 6. But previously to the interpolating the law of mortality from tables of experience, I will premise that if, according to our notation, the number of living at the age  $x$  be denoted by  $L_x$ , and  $\lambda$  be the characteristic of a logarithm, or such that  $\lambda(L_x)$  may denote the logarithm of that number, that if  $\lambda(L_a) - \lambda(L_{a+r}) = m$ ,  $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = mp$ ,  $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = m^2p$ ; and generally  $\lambda(L_{a+n-r}) - \lambda(L_{a+n}) = m \cdot p^{\frac{n}{r}-1}$ ; that by continual addition we shall have  $\lambda(L_a) - \lambda(L_{a+n}) = m(1 + p + p^2 + p^3 + \dots + p^{\frac{n}{r}-1}) = m \cdot \frac{1-p^{\frac{n}{r}}}{1-p}$ ; and therefore if  $p^{\frac{1}{r}} = q$ , and  $\epsilon$  be put equal to the number whose common logarithm is  $\frac{m}{1-q^n}$ , we shall have  $\lambda(L_{a+n}) = \lambda(L_a) - \lambda(\epsilon) \times (1 - q^n) = \lambda\left(\frac{L_a}{\epsilon}\right) + \lambda(\epsilon) \cdot q^n$ ;  $\therefore L_{a+n} = \frac{L_a}{\epsilon} \times \epsilon^{q^n}$ ; and this equation, if for  $a+n$  we write  $x$ , will give  $L_x = \frac{L_a}{\epsilon} \cdot \epsilon^{q^{-a} \times q^x}$ ; and consequently if  $\frac{L_a}{\epsilon}$  be put

$= d$ , and  $\bar{\epsilon}^{q^{-a}} = g$ , the equation will stand  $L_x = d \cdot \bar{g}^{q^x}$ , and  $\lambda(g) = \lambda(\epsilon) \times q^{-a} = \frac{m q^{-a}}{1 - q^r}$ ; and I observe that when  $q$  is affirmative, and  $\lambda(\epsilon)$  negative, that  $\lambda(g)$  is negative. The equation  $L_x = d \cdot \bar{g}^{q^x}$  may be written in general  $\lambda(L_x) = \lambda(d) \pm$  the positive number whose common logarithm is  $\{\lambda^2(g) + x \lambda(g)\}$ , the upper or under sign to be taken according as the logarithm of  $g$  is positive or negative,  $\lambda^2$  standing for the characteristic of a second logarithm; that is, the logarithm of a logarithm,  $\lambda(q) = \frac{1}{r} \times \lambda(p)$ ,  $\lambda^2(g) = \lambda^2(\epsilon) - a \cdot \lambda(q) = \lambda\left(\frac{m}{1-p}\right) - a \cdot \lambda(q) = \lambda(m) - \lambda(1-p) - a \lambda(q)$ ; also  $\lambda(d) = \lambda(L_a) - \frac{m}{1-p}$ .

Art. 7. Applying this to the interpolation of the Northampton table, I observe that taking  $a = 15$  and  $r = 10$  from that table, I find  $\lambda(L_a) - \lambda(L_{a+r}) = ,0566 = m$ ,  $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = ,0745$ ,  $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = ,0915$ , and  $\lambda(L_{a+3r}) - \lambda(L_{a+4r}) = ,1228$ ; now if these numbers were in geometrical progression, whose ratio is  $p$ , we should have respectively  $m = ,0566$ ;  $mp = ,0745$ ;  $mp^2 = ,0915$ ;  $mp^3 = ,1228$ . No value of  $p$  can be assumed which will make these equations accurately true; but the numbers are such that  $p$  may be assumed, so that the equation shall be nearly true; for resuming the first and last equations we have  $p^3 = \frac{1228}{566}$ ;  $\therefore$  logarithm of  $p = \frac{1}{3}(\logarithm of 1228 - \logarithm of 566) = ,11213$ ,  $\therefore \lambda(q) = ,011213$  and  $p = 1.2944$ . And to examine how near this is to the thing required, continually to the logarithm of ,0566 namely ,275282, adding ,11213 which is the logarithm of  $p$ , we have respectively for the

logarithms of  $mp$ , of  $mp^2$ , of  $mp^3$  the values  $\bar{2},8649$ ,  $\bar{2},9771$ ,  $\bar{1},0892$ ; the numbers corresponding to which are  $,07327$ ;  $,09486$ ;  $,1228$ ; and consequently  $m$ ,  $mp$ ,  $mp^2$ , and  $mp^3$  respectively equal to  $,0566$ ;  $,07327$ ;  $,09486$ , and  $,1228$  which do not differ much from the proposed series  $,0566$ ;  $,07327$ ;  $,09486$ , and  $,1228$ ; and according to our form for interpolation, taking  $m = ,0566$  and  $p = 1,2944$ ; we have  $\frac{m}{1-p} = -\frac{,0566}{,2944} = -,1922$ ; and  $\lambda(L_{15})$  agreeably to the Northampton tables, being  $= 3,7342$  we have  $\lambda(d) = 3,7342 + ,1922 = 3,9264$ ,  $d = 8441$ ,  $\lambda^2(q)$ , that is to say, the logarithm of the logarithm of  $q = \lambda\left(\frac{m}{1-p}\right) - a\lambda(q) = \bar{1},28375 - ,16819 = \bar{1},1156$ ,  $\lambda(g) = - ,130949 = \bar{1},8695$ , the negative sign being taken because  $\lambda(g) = \lambda(\epsilon) \times q^{-a} = \frac{m}{1-q} \cdot q^{-a}$ , and  $g = ,7404$ . And therefore  $x$  being taken between the limits, we are to examine the degree of proximity of the equation  $L_x = 8441 \times \sqrt[1,0261^x]{7404}$  or  $\lambda(L_x)$ , that is, the logarithm of the number of living at the age  $x = 3,9264$  — number whose logarithm is  $(\bar{1},11556 + x \times .011213)$ , as the logarithm of  $g$  is negative. The table constructed according to this formula, which I shall lay before the reader, will enable him to judge of the proximity it has to the Northampton table; but previously thereto shall show that the same formula, with different constants, will serve for the interpolations of other tables.

Art. 8. To this end let it be required to interpolate DEPARCIEUX's tables, in Mr. BAILY's life annuities, between the ages 15 and 55.

The logarithms of the living at the age of

15	are 2,92840 differences	$=,03966 = \lambda(L_{15}) - \lambda(L_{25})$
25	2,88874	$,04738 = \lambda(L_{25}) - \lambda(L_{35})$
35	2,84136	$,04757 = \lambda(L_{35}) - \lambda(L_{45})$
45	2,79379	$,07280 = \lambda(L_{45}) - \lambda(L_{55})$
55	2,72099	

Here the three first differences, instead of being nearly in geometrical progression are nearly equal to each other, showing from a remark above, that the living, according to these tables, are nearly in geometrical progression ; and the reader might probably infer that this table will not admit of being expressed by a formula similar to that by which the Northampton table has been expressed between the same limits, but putting,

on the supposition of the possibility, though the thing cannot be accurately true,

$$\left\{ \begin{array}{l} \lambda(L_{15}) = \dots \dots \dots \dots \dots = 2,92840 \\ \lambda(L_{25}) = \lambda(L_{15}) - m \dots \dots \dots = 2,88874 \\ \lambda(L_{35}) = \lambda(L_{15}) - m - mp \dots \dots = 2,84136 \\ \lambda(L_{45}) = \lambda(L_{15}) - m - mp - mp^2 \dots = 2,79379 \\ \lambda(L_{55}) = \lambda(L_{15}) - m - mp - mp^2 - mp^3 = 2,72099 \end{array} \right\} \text{and we shall have}$$

$\lambda(L_{15}) - \lambda(L_{35})$  or its equal  $m + mp = ,08704$ , and  $\lambda(L_{35}) - \lambda(L_{55})$  or its equal  $p^2 \times \overline{m + pm} = ,12037$ ;  $\therefore p^2 = \frac{12037}{8704}$  and the log. of  $p = \frac{\log. \text{ of } 12037 - \log. \text{ of } 8704}{2} = ,0703997$  and  $p = 1,176$ ,  $m = \frac{,08704}{1 + p} = \frac{,08704}{2,176} = ,04$ . And to see how these values of  $m$  and  $p$  will answer for the approximate determination of the logarithms above set down of the numbers of living at the ages 15, 25, 35, 45, and 55, we have the following easy calculation by continually adding the logarithm of  $p$

Logarithm of $m = \bar{2},6020600$	$\lambda(L_{15}) = 2,92840$
Log. of $p = 0,0703997$	$-m = -,04$
	therefore $mp = ,047039$
Log. of $mp = \bar{2},6724597$	$\lambda(L_{25}) = 2,88840$
Log. of $mp^2 = 2,7428594$	$-mp = -,04704$
Log. of $mp^3 = 2,8128591$	$2,84136$
	$-mp^2 = -,05532$
	$2,78604$
	$-mp^3 = -,06505$
	$2,72099$

These logarithms of the approximate number of living at the ages 15, 25, 35, 45 and 55, are extremely near those proposed, and the numbers corresponding to these give the number of living at the ages 15, 25, 35, 45 and 55, respectively, 848; 773,4; 694; 612,3; and 526; differing very little from the table in Mr. BAILY's life annuities; namely, 848; 774; 694; 622 and 526. And we have  $a = 15$ ,  $r = 10$ ,  $m = ,04$ ;  $\lambda(m) = \bar{2},60206$ ;  $1 - p = -,176$ ;  $\lambda q = \frac{1}{10} \lambda(p) = ,00703997$ ;  $\lambda(g) = \frac{m q^{-a}}{1-p} = -\frac{,04 \times q^{-a}}{,176}$ , and is negative;  $\lambda \lambda(g) = \lambda(.04) - 15 \times ,00704 - \lambda(.176) = 1,25095$ ;  $\lambda(d) = \lambda(L_a) - \frac{m}{1-p} = 2,9284 + ,22727 = 3,1557$ ;  $\therefore \lambda(L_x) = 3,1557$  — number whose log. is  $(1,25095 + ,00704 x)$ , for the logarithm of living in DEPARCIEUX' table in Mr. BAILY's annuities, between the limits of age 15 and 55. The table which we shall insert will afford an opportunity of appreciating the proximity of this formula to the table.

Art. 9. To interpolate the Swedish mortality among males between the ages of 10 and 50, from the table in Mr. BAILY's annuities:

Here  $\lambda(L_{10}) = 3,779091$

$$\lambda(L_{20}) = 3,746868 \text{ to be assumed} = \lambda(L_{10}) - m$$

$$\lambda(L_{30}) = 3,703205 \quad \dots \quad = \lambda(L_{10}) - m - mp$$

$$\lambda(L_{40}) = 3,648165 \quad \dots \quad = \lambda(L_{10}) - m - mp - mp^2$$

$$\lambda(L_{50}) = 3,564192 \quad \dots \quad = \lambda(L_{10}) - m - mp - mp^2 - mp^3$$

Consequently  $m + mp = \lambda(L_{10}) - \lambda(L_{30}) = ,075886$ , and  $\lambda(L_{30}) - \lambda(L_{50}) = p^2 \times \frac{m + mp}{m} = ,139013$ ; therefore  $p^2 = \frac{139013}{75886}$ , and  $\lambda(p) = ,1314468$ ;  $\therefore p = 1,3535$ ;  $m = \frac{,075886}{1 + p} = \frac{,075886}{2,3535}$ ;  $\lambda(m) = \frac{1}{2},5084775$ ;  $m = ,032244$ ;  $a = 10$ ;  $r = 10$ ;  $\lambda(q) = ,01314468$ ;  $\lambda(g) = \frac{m \cdot q^{-10}}{,3535}$ , negative;  $\lambda(g) = \lambda(m) - 10 \lambda(q) - \lambda(,3535) = \frac{1}{2},82861$ ;  $\lambda(d) = \lambda(L_a) - \frac{m}{1 - p} = 3,779091 + ,091218, = 3,8703$ ; consequently this will give between the ages 10 and 50 of Swedish males,

$\lambda(L_x)$  or the logarithm of the living at the age of  $x = 3,8703 -$  number, whose logarithm is  $(\frac{1}{2},82861 + ,013145 x)$ .

A table will also follow to show the proximity of this with Mr. BAILY's table.

Art. 10. For Mr. MILNE's table of the Carlisle mortality we have, as given by that ingenious gentleman,

$$\lambda(L_{10}) = 3,81023$$

$$\lambda(L_{20}) = 3,78462$$

$$\lambda(L_{30}) = 3,75143$$

$$\lambda(L_{40}) = 3,70544$$

$$\lambda(L_{50}) = 3,64316$$

$$\lambda(L_{60}) = 3,56146$$

And the difference of these will form a series nearly in geometrical progression, whose common ratio is  $\frac{4}{3}$ , and in consequence of this, the first method may be adopted for the

interpolations. Thus because  $\lambda(L_{10}) - (L_{20}) = ,02561$ , the first term of the differences, and  $\lambda(L_{50}) - \lambda(L_{60}) = ,0817$ , the fifth term of the differences: take the common ratio  $= \frac{817}{256}^{\frac{1}{4}}$ , and  $m = ,0256$ ;  $\therefore \lambda(m) = \frac{1}{2},40824$ . These will give  $\lambda(p) = ,126$ ;  $p = 1,3365$ ;  $a = 10$ ,  $r = 10$ ,  $\lambda(q) = ,0126$ ,  $\lambda(\epsilon) = \frac{m}{1-p} = -\frac{,0256}{,3365}$ ;  $\therefore \lambda(g)$  negative;  $\lambda \lambda g = 2,40824 - \lambda(,3365) - ,126 = \frac{1}{2},75526$ ; and  $\lambda(d) = \lambda(L_{10}) + \frac{,0256}{,3365} = 3,88631$ , and accordingly, to interpolate the Carlisle table of mortality for the ages between 10 and 60, we have for any age  $x$ ,

$$\lambda(L_x) = 3,88631 - \text{number whose logarithm is } (\frac{1}{2},88126 + ,0126 x).$$

Here we have formed a theorem for a larger portion of time than we had previously done. If by the second method the theorem should be required from the data of a larger portion of life, we must take  $r$  accordingly larger; thus if  $a$  be taken 10,  $r = 12$ , then the interpolation would be formed from an extent of life from 10 to 58 years; and referring to Mr. MILNE's tables, our second method would give  $\lambda(L_x) = 3,89063 - \text{the number whose logarithm is } (\frac{1}{2},784336 + ,0120948 x)$ ; this differs a little from the other, which ought to be expected.

If the portion between 60 and 100 years of Mr. MILNE's Carlisle table be required to be interpolated by our second method, we shall find  $p = 1,86466$ ;  $\lambda(m) = 1,30812$ ;  $m = ,20329$ , &c. and we shall have  $\lambda(L_x) = 3,79657 - \text{the number whose logarithm is } (\frac{1}{2},74767 + ,02706 x)$ .

This last theorem will give the numbers corresponding to the living at 60, 80, and 100, the same as in the table; but for the ages 70 and 90, they will differ by about one year:

the result for the age of 70 agreeing nearly with the living corresponding to the age 71; and the result for the age 90, agreeing nearly with the living at the age 89 of the Carlisle tables.

Art. 11. Lemma. If according to a certain table of mortality, out of  $a$ , persons of the age of 10, there will arrive  $b, c, d, \&c.$  to the age 20, 30, 40, &c.; and if according to the tables of mortality, gathered from the experience of a particular society, the decrements of life between the intervals 10 and 20, 20 and 30, 30 and 40, &c. is to the decrements in the aforesaid table between the same ages, proportioned to the number of living at the commencement of those intervals respectively, as 1 to  $n$ , 1 to  $n'$ , 1 to  $n''$ , &c. it is required to construct a table of mortality of that society, or such as will give the above data.

Solution. According to the first table, the decrements of life from 10 to 20, 20 to 30, 30 to 40, &c. respectively, will be found by multiplying the number of living at the commencement of each period by  $\frac{a-b}{a}$ ,  $\frac{b-c}{b}$ ,  $\frac{c-d}{c}$ , &c., and therefore, in the Society proposed, the corresponding decrements will be found by multiplying the number of living at those ages by  $\frac{a-b}{a} n$ ;  $\frac{b-c}{b} n'$ ;  $\frac{c-d}{c} n''$  &c.; and the number of persons who will arrive at the ages 20, 30, 40, &c. will be the numbers respectively living at the ages 10, 20, 30, &c. multiplied respectively by  $\frac{1-n \cdot a + nb}{a}$ ,  $\frac{1-n' \cdot b + nc}{b}$ ,  $\frac{1-n'' \cdot c + n'd}{c}$ , &c.; hence out of the number  $a$ , living at the age 10, there will arrive at the age 10, 20, 30, 40, 50, &c. the numbers  $\frac{1-n \cdot a + nb}{1-n \cdot a + nb} \times \frac{1-n' \cdot b + nc}{b}$ ;  $\frac{1-n \cdot a + nb}{1-n \cdot a + nb} \times \frac{1-n' \cdot b + nc}{b} \times \frac{1-n'' \cdot c + n'd}{c}$ ; &c. and the numbers for the intermediate ages must be found by interpolation.

In the ingenious Mr. MORGAN's sixth edition of PRICE's Annuities, p. 183, vol. i. it is stated, that in the Equitable Assurance Society, the deaths have differed from the Northampton tables; and that from 10 to 20, 20 to 30, 30 to 40, 40 to 50, 50 to 60, and 60 to 80, it appears that the deaths in the Northampton tables were in proportion to the deaths which would be given by the experience of that society respectively, in the ratios of 2 to 1; 2 to 1; 5 to 3; 7 to 5, and 5 to 4. According to this, the decrements in 10 years of those now living at the ages 10, 20, 30, and 40, will be the number living at those ages multiplied respectively by ,0478; ,0730; ,1024; ,1284; and the deaths in twenty years of those now living at the age of 60, would be the number of those living multiplied by ,3163. And also, taking, according to the Northampton table, the living at the age of 10 years equal to 5675, I form a table for the number of persons living at

the ages . . .	10	20	30	40	50	60	70	80
being . . .	5675	5403.5	5010	4496	3919	3116	* 1197	
and the log. of the number of persons living }	3,75612	3,73268	3,69984	3,65283	3,59318	3,49360	*	

Consequently, if  $a = 20$ ,  $r = 10$ , we have  $\lambda(L_{20}) = 3,73268$ ;  $\lambda(L_{40}) = \lambda(L_{20}) - m - mp = 3,65283$ ;  $\lambda(L_{60}) = L_{20} - m - mp - mp^2 - mp^3 = 3,49360$ ;  $m \cdot 1 + p = ,07985$ ; and  $mp^2 \times 1 + p = 3,65283 - 3,49360 = ,15923$ ; hence  $\lambda(p) = \frac{1}{2} \lambda\left(\frac{,15923}{,07985}\right) = ,149875$ ; and  $p = 1,412131$ ;  $\lambda(m) = \lambda(,07985) - \lambda(,149875) = ,519874$ ; and  $m = ,033013$ ;  $\therefore \lambda(\epsilon) = \frac{-m}{,412131}$  negative;  $\therefore \lambda(g)$  is negative;  $\lambda \lambda(g) = \lambda m - \lambda,412131 - ,0149875 \times 20 = ,6051$ ;  $\lambda(d) = \lambda(L_{20}) - \lambda(\epsilon) = 3,73268 - ,080302 = 3,813$  sufficiently near; and our formula for the

mortality between the ages of 20 and 60, which appears to me to be the experience of the Equitable Society, is  $\lambda(L_x) = 3,813$  — the number whose log. is  $(-2.6051 + ,0149875x)$ .

This formula will give

At the ages	10	20	30	40	50	60	70	80
No. of living	5703,2	5403,5	5007	4496	3862	3116	*	1500
Differs from the proposed by	{ 28,2	0	+ 3	0	- 57	0	*	303

In the table of Art. 12, the column marked 1, represents the age; column marked 2, represents the number of persons living at the corresponding age; column marked 3, the error to be added to the number of living deduced from the formula, to give the number of living of the table for which the formula is constructed; column marked 4, gives the error in age, or the quantity to be added to the age in column 1, that would give the number of living in the original table, the same as in column 2. It may be proper to observe, that where the error in column 3 and 4 is stated to be 0, it is not meant to indicate that a perfect coincidence takes place, but that the difference is too small to be worth noticing.

Art. 12.  $\lambda(L) = \lambda(d)$  — number whose logarithm is  $(\lambda^2(g) + x \lambda q)$ .

Northampton.				Deparcieux.			Sweden.			Carlisle.			Formula of supposed experience of the Equitable,					
1	2	3	4	2	3	4	2	3	4	2	3	4	2	3	2	3	4	1
10							6013	0		6460	0	0	5703	-28	6460			
11							5974	-16	$\frac{1}{3}$	6427	+4	$\frac{1}{3}$	5677		6431			
12							5935	-22	$\frac{1}{2}$	6393	+7	$\frac{1}{2}$	5650		6400			
13							5894	-26	$\frac{1}{3}$	6358	+10	$\frac{1}{3}$	5622		6368 $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	
14							5852	-24 $\frac{1}{2}$	$\frac{8}{3}$	6322	+13	$\frac{1}{3}$	5594		6336	-1	$\frac{1}{3}$	14
15	5423			848	0	0	5810	-22 $\frac{1}{2}$	$\frac{3}{8}$	6286	+14	$\frac{1}{3}$	5564		6302	-2	$\frac{1}{2}$	15
16	5360	+13	$\frac{1}{2}$	841	+1	$\frac{1}{6}$	5767	-18	$\frac{1}{3}$	6248	+13	$\frac{1}{3}$	5534		6268 $\frac{1}{2}$	-7 $\frac{1}{2}$	$\frac{1}{6}$	16
17	5297	+23	$\frac{1}{2}$	833 $\frac{1}{2}$	+1 $\frac{1}{2}$	$\frac{1}{4}$	5722	-12	$\frac{1}{3}$	6210	+9	$\frac{1}{3}$	5503		6233	-14	$\frac{1}{3}$	17
18	5233	+29	$\frac{1}{2}$	826	+2	$\frac{1}{3}$	5677	-6	$\frac{1}{7}$	6171	+5	$\frac{1}{3}$	5470		6196 $\frac{1}{2}$	-20 $\frac{1}{2}$	$\frac{1}{2}$	18
19	5168	+31	$\frac{1}{2}$	819	+2	$\frac{1}{3}$	5630	-3	$\frac{1}{14}$	6131	+2	$\frac{1}{2}$	5437		6159	-26	$\frac{1}{3}$	19
20	5102	+30	$\frac{1}{2}$	811	+3	$\frac{1}{3}$	5583	-0	0	6090	0	0	5403	o	6120 $\frac{1}{2}$	-30 $\frac{1}{2}$	$\frac{5}{6}$	20
21	5036	+24	$\frac{1}{2}$	804	+2	$\frac{1}{2}$	5534	-1	0	6048	-1	0	5368		6081	-34	$\frac{1}{6}$	21
22	4969	+16	$\frac{1}{5}$	796	+2	$\frac{1}{4}$	5484	-1	0	6005	+0	0	5333		6040 $\frac{1}{2}$	-35 $\frac{1}{2}$	$\frac{5}{6}$	22
23	4899	+11	$\frac{1}{7}$	789	+1	$\frac{1}{5}$	5434	-1	0	5962	+1	0	5295		5998 $\frac{1}{2}$	-35 $\frac{1}{2}$	$\frac{5}{6}$	23
24	4830	+5	$\frac{1}{13}$	781	+1	$\frac{1}{3}$	5382	-4	$\frac{1}{8}$	5917	+4	$\frac{1}{10}$	5258		5955 $\frac{1}{2}$	-34 $\frac{1}{2}$	$\frac{5}{6}$	24
25	4762	-2	$\frac{1}{7}$	774	+0	0	5329	-6	$\frac{1}{9}$	5870 $\frac{1}{2}$	+8 $\frac{1}{2}$	$\frac{1}{5}$	5218		5911	-32	$\frac{5}{6}$	25
26	4689	-4	$\frac{1}{9}$	766	+0	0	5275	-7	$\frac{1}{3}$	5825	+11	$\frac{1}{4}$	5178		5866	-30	$\frac{1}{7}$	26
27	4616	-6	$\frac{1}{3}$	757	+1	$\frac{1}{8}$	5220	-7	$\frac{1}{8}$	5777	+16	$\frac{1}{4}$	5137		5819	-26	$\frac{5}{9}$	27
28	4545	-10	$\frac{1}{5}$	750	+0	0	5164	-6	$\frac{1}{5}$	5729	+19	$\frac{1}{8}$	5095		5771	-23	$\frac{1}{5}$	28
29	4472	-12	$\frac{1}{6}$	742	+0	0	5107	-4	$\frac{1}{14}$	5679	+19	$\frac{1}{3}$	5051		5722	-24	$\frac{1}{3}$	29
30	4403	-22	$\frac{1}{3}$	734	+0	0	5049	-0	0	5628 $\frac{1}{2}$	+13 $\frac{1}{2}$	$\frac{1}{4}$	5007	-3	5671 $\frac{1}{2}$	-29 $\frac{1}{2}$	$\frac{4}{5}$	30
31	4325	-15	$\frac{1}{5}$	726	+0	0	4989 $\frac{1}{2}$	-1	$\frac{1}{2}$	5578	+7	$\frac{1}{3}$	4961		5620	-34	$\frac{1}{5}$	31
32	4250	-15	$\frac{1}{5}$	718	+0	0	4929	-1	0	5524	+4	$\frac{1}{4}$	4914		5567	-39	$\frac{5}{6}$	32
33	4174	-14	$\frac{1}{7}$	710	+0	0	4868	-0	0	5470 $\frac{1}{2}$	+1 $\frac{1}{2}$	$\frac{1}{3}$	4866		5512 $\frac{1}{2}$	-42 $\frac{1}{2}$	$\frac{4}{3}$	33
34	4098	-13	$\frac{1}{5}$	702	+0	0	4805	+3	$\frac{1}{20}$	5416	+1	0	4817		5456 $\frac{1}{2}$	-39 $\frac{1}{2}$	$\frac{5}{7}$	34
35	4021	-11	$\frac{1}{7}$	694	+0	0	4741	+7	$\frac{1}{9}$	5360	+2	$\frac{1}{7}$	4767		5399	-37	$\frac{1}{5}$	35
36	3944	-11	$\frac{1}{7}$	686	+0	0	4676	+12	$\frac{1}{5}$	5303	+4	$\frac{1}{4}$	4715		5341	-34	$\frac{1}{4}$	36
37	3866	-6	$\frac{1}{2}$	678	+0	0	4611	+17	$\frac{1}{4}$	5245 $\frac{1}{2}$	+5 $\frac{1}{8}$	$\frac{1}{5}$	4662		5281	-30	$\frac{1}{7}$	37
38	3788	-3	$\frac{1}{27}$	669	+2	$\frac{1}{4}$	4544	+24	$\frac{1}{5}$	5187	+7	$\frac{1}{8}$	4608		5219 $\frac{1}{2}$	-25 $\frac{1}{2}$	$\frac{4}{5}$	38
39	3709	+1	$\frac{1}{75}$	661	+3	$\frac{1}{3}$	4476	+28	$\frac{1}{2}$	5127	+9	$\frac{1}{6}$	4553		5157	-21	$\frac{1}{3}$	39
40	3630	+5	$\frac{1}{5}$	653	+4	$\frac{1}{2}$	4407	+41	$\frac{2}{3}$	5065	+10	$\frac{1}{6}$	4496		5093	-18	$\frac{1}{4}$	40
41	3551	+8	$\frac{1}{9}$	645	+5	$\frac{1}{12}$	4337	+46	$\frac{1}{3}$	5004	+5	$\frac{1}{13}$	4438		5027	-18	$\frac{1}{4}$	41
42	3473	+11	$\frac{1}{7}$	636	+7	$\frac{1}{1}$	4266	+45	$\frac{1}{5}$	4941	-1	0	4379		5960	-20	$\frac{1}{7}$	42
43	3392	+12	$\frac{1}{6}$	628	+8	$\frac{1}{1}$	4194	+37	$\frac{1}{6}$	4877	-8	$\frac{1}{9}$	4319		4892	-23	$\frac{1}{5}$	43
44	3312	+14	$\frac{1}{6}$	619	+10	$\frac{1}{4}$	4121	+30	$\frac{1}{8}$	4812	-14	$\frac{1}{5}$	4257		4882	-24	$\frac{1}{3}$	44
45	3235	+13	$\frac{1}{6}$	611	+11	$\frac{1}{5}$	4047	+24	$\frac{1}{10}$	4746	-19	$\frac{1}{4}$	4194		4751	-24	$\frac{1}{3}$	45
46	3152	+18	$\frac{1}{4}$	603	+12	$\frac{1}{2}$	3973	+18	$\frac{1}{5}$	4678	-21	$\frac{1}{10}$	4130		4678 $\frac{1}{2}$	-21 $\frac{1}{2}$	$\frac{4}{5}$	46
47	3072	+20	$\frac{1}{4}$	594	+13	$\frac{1}{12}$	3897	+14	$\frac{1}{6}$	4610	-22	$\frac{1}{9}$	4065		4604 $\frac{1}{2}$	-16 $\frac{1}{2}$	$\frac{4}{5}$	47
48	2991	+23	$\frac{1}{3}$	586	+13	$\frac{1}{12}$	3821	+10	$\frac{1}{8}$	4541	-20	$\frac{1}{10}$	3998		4529	-8	$\frac{1}{8}$	48
49	2911	+25	$\frac{1}{3}$	577	+13	$\frac{1}{12}$	3744	+7	$\frac{1}{12}$	4471	-13	$\frac{1}{5}$	3931		4452 $\frac{1}{2}$	+5 $\frac{1}{2}$	$\frac{1}{1}$	49
50	2831	+26	$\frac{1}{3}$	569	+12	$\frac{1}{2}$	3660	0	0	4400 $\frac{1}{2}$	-3 $\frac{1}{2}$	$\frac{1}{7}$	3862	+57	4375	+22	$\frac{1}{3}$	50
51	2752	+24	$\frac{1}{3}$	560	+11	$\frac{1}{2}$	3587	-16	$\frac{1}{6}$	4329	+9	$\frac{1}{7}$	3793		4296 $\frac{1}{2}$	+31 $\frac{1}{2}$	$\frac{5}{6}$	51
52	2672	+22	$\frac{1}{4}$	552	+8	$\frac{1}{1}$	3508	-32	$\frac{1}{3}$	4256	+20	$\frac{1}{3}$	3721		4215	+61	$\frac{1}{1}$	52
53	2593	+19	$\frac{1}{4}$	543	+6	$\frac{3}{4}$	3428	-47	$\frac{1}{2}$	4182	+29	$\frac{1}{7}$	3649		4133	+78	$\frac{1}{7}$	53
54	2514	+16	$\frac{1}{5}$	536	+2	$\frac{1}{4}$	3348	-62	$\frac{2}{3}$	4108	+35	$\frac{1}{2}$	3575		4050	+93	$\frac{1}{4}$	54
55	2430	+12	$\frac{1}{7}$	526	+0	0				4033	+40	$\frac{1}{4}$	3501		3966	+107	$\frac{1}{3}$	55
56	2358	+8	$\frac{1}{10}$							3957	+43	$\frac{1}{7}$	3426		3881	+219	$\frac{1}{2}$	56
57	2280	+4	$\frac{1}{20}$							3880	+44	$\frac{1}{5}$	3350		3794 $\frac{1}{2}$			57
58	2206	-4	$\frac{1}{20}$							3803	+39	$\frac{1}{3}$	3273		3706			58
59	2123	-3	$\frac{1}{30}$							3724	+25	$\frac{1}{4}$	3195		3169			59
60	2052	-14	$\frac{1}{5}$							3646	-3	0	3116	o	3529	not compared.	not compared.	60
	$\lambda(d) = 3,9265$	$\lambda(d) = 3,1557$	$\lambda(d) = 3,8703$				$\lambda(d) = 3,8063$	$\lambda(d) = 3,813$							$\lambda(d) = 3,86743$			
	$\lambda^2(g) = 1,11556$	$\lambda^2(g) = 1,25095$	$\lambda^2(g) = 2,82861$				$\lambda^2(g) = 2,784336$	$\lambda^2(g) = 2,6051$							$\lambda^2(g) = 2,6051$			
	$\lambda(q) = 0,011213$	$\lambda(q) = 0,00704$	$\lambda(q) = 0,0131457$				$\lambda(q) = 0,0120948$	$\lambda(q) = 0,0149875$							$\lambda(q) = 0,0149875$			

## CHAPTER II.

ARTICLE 1. The near proximity to the geometrical progression of the series expressing the number of persons living at equal small successive intervals of time during short periods, out of a given number of persons living at the commencement of those intervals, affords a very convenient mode of calculating values connected with life contingencies, for short limited periods; by offering a manner of forming general tables, applicable (by means of small auxiliary tables of the particular mortalities) to calculations for any particular mortality; and by easy repetition, to calculate the values for any length of period for any table of mortality we please.

If, for instance, it were required to find the value of an annuity of an unit for  $p$  years, on three lives of the age  $b, c, d$ , the rate of interest being such that the present value of an unit to be received at the expiration of one year, be equal to  $r$ , then the

value of the first payment would be  $\frac{L_{b+1}}{L_b} \times \frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} \times r$ ;

and of the  $p^{\text{th}}$  payment the present value would be  $\frac{L_{b+p}}{L_b} \times \frac{L_{c+p}}{L_c} \times \frac{L_{d+p}}{L_d} \times r^p$ ; but if  $L_{b+p} = L_b \times \left(\frac{L_{b+1}}{L_b}\right)^p$  whether  $p$  be

1, 2, 3, &c. which will be the case when  $L_b, L_{b+1}, L_{b+2}$ , &c. form a geometrical progression, and similarly, if  $L_{c+p} = L_c \times \left(\frac{L_{c+1}}{L_c}\right)^p$ , and also,  $L_{d+p} = L_d \times \left(\frac{L_{d+1}}{L_d}\right)^p$ , the pre-

sent value of the  $p^{\text{th}}$  payment will be  $\left(\frac{L_1 : b, c, d}{L_{b, c, d}} r\right)^p$ ; hence, if  $\frac{L_1 : b, c, d}{L_{b, c, d}} r$  be put =  $a$ , the value of the annuity will be  $a + a^2 + a^3 + a^4 \dots a^p = \frac{a - a^{p+1}}{1 - a} = \frac{1 - a^p}{a^{p+1} - 1}$ .

Art. 2. Consequently, let a general table be formed of the logarithm of  $\frac{1 - a^p}{a^{p+1} - 1}$  for every value of the log. of  $a^p$ ; and also let a particular table be formed for every value of the log. of  $\frac{L_x + p}{L_x}$  according to the particular table of mortality to be adopted; from the last table take the log. of  $\frac{L_b + p}{L_b}$ ,  $\frac{L_c + p}{L_c}$ ,  $\frac{L_d + p}{L_d}$ ; and also from a table constructed for the purpose, take the log. of  $r^p$ , add these four logs. together, and the sum will be the log. of  $\bar{a}^p$ , which being sought for in the general table, will give the log. of  $\left(\frac{1 - a^p}{a^{p+1} - 1}\right)$  which will be the log. of the annuity sought for the term  $p$ , on supposition of the geometrical progression being sufficiently near. Here I remark, that were it not for more general questions than the above, it would be preferable to have general tables formed for the values of  $\frac{1 - a^p}{a^{p+1} - 1}$ , instead of the log. of such values; but from the consideration that for most purposes a table of the logs. of  $\frac{1 - a^p}{a^{p+1} - 1}$  will be found most convenient, I have had them calculated in preference.

Art. 3. The shorter the periods are, the nearer does the series of the number of persons living at the equal intervals of successive ages approximate to the geometrical progression; and consequently this mode, by the assumption of sufficiently short periods, and frequent repetitions, will answer

for any degree of accuracy the given table of mortality will admit of, but then the labour will be increased in proportion.

Art. 4. There are different modes of obviating, in a great measure, this inconvenience, by assuming an accommodated ratio for the given age, instead of the real ratio, from amongst which I shall only for the present select a few. The first is as follows: find for every value of  $a$ , the log. of  $\frac{\frac{1}{p}L_x+y}{L_x}$ , that

is, the log. of  $\frac{L_{x+1}}{L_x} + \frac{L_{x+2}}{L_x} + \frac{L_{x+3}}{L_x} \dots \frac{L_{x+p}}{L_x}$ ; seek this value in the general table, which will give the corresponding value of the log. of  $a^p$ ; and construct a table of such values for every value of  $x$ , and adopt these values for log. of  $a^p$ , instead of the abovenamed values of the log. of  $\frac{L_{x+p}}{L_x}$ , for the determination of the values of the limited periods: the preference of this to the first proposed method consists in this; that if the series  $\frac{1}{L_b} \times (L_{b+1} + L_{b+2} + L_{b+3} \dots L_{b+p}) = \epsilon + \epsilon^2 + \epsilon^3 + \&c. \dots \epsilon^p$ , the series  $\frac{L_{b+1}}{L_b}, \frac{L_{b+2}}{L_b}, \&c.$  being nearly in geometrical progression, and  $\frac{L_{b+1}}{L_b} - \epsilon = \epsilon_1, \frac{L_{b+2}}{L_b} - \epsilon^2 = \epsilon_2, \&c. \epsilon_1, \epsilon_2, \&c.$  will be small, and  $\epsilon_1 + \epsilon_2 + \epsilon_3 \dots \epsilon_p = 0$ , and there-

fore, if the series  $\frac{L_{c+1}}{L_c}, \frac{L_{c+2}}{L_c}, \frac{L_{c+3}}{L_c}, \&c.$  and  $\frac{L_{d+1}}{L_d}, \frac{L_{d+2}}{L_d}, \&c.$  formed accurately geometrical progressions, and the value of  $\frac{L_{c+1} \times L_{d+1}}{L_c \times L_d} \cdot r = m$ , the value of the annuity for the term, would be accurately equal to  $m \epsilon + m^2 \epsilon^2 + m^3 \epsilon^3 \dots + m^p \epsilon^p + m \epsilon_1 + m^2 \epsilon_2 + m^3 \epsilon_3 \dots + m^p \epsilon_p$ , but because in

general  $\frac{L_{c+1}}{L_c}$ ,  $\frac{L_{d+1}}{L_d}$  and  $r$  differ very little from unity,  $m$  will not differ much from unity; and therefore if  $p$  be not great,  $m$ ,  $m^2$ ,  $m^3$ , &c. will not differ much from unity; and consequently, as  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$ , &c. are small,  $m\epsilon_1 + m^2\epsilon_2 + m^3\epsilon_3 + \dots + m^p\epsilon_p$  will not differ much from  $\epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_p$ ; but this has been shown to be 0; consequently  $m\epsilon_1 + m^2\epsilon_2 + m^3\epsilon_3 + \dots + m^p\epsilon_p$  differs very little from 0, or in other words is very small; and consequently, the value of the annuity differs very little from  $m\epsilon + m^2\epsilon^2 + m^3\epsilon^3 + \dots + m^p\epsilon^p$ ; and the same method of demonstration would apply with any one of the other ages, the remaining ages being supposed to possess the property of the accurate geometrical progression; notwithstanding this, however, as none of them probably will contain that property, but in an approximate degree, a variation in the above approximations may be produced of a small quantity of the second order; that is, if the order of the product of two small quantities; but, as in this approximation, I was only aiming at retaining the quantities of the first order, I do not consider this as affecting the result as far as the approximation is intended to reach: thus far with regard to the first accommodated ratios.

Art. 5. Moreover, on the supposition that  $L_c$ ,  $L_{c+1}$ ,  $L_{c+2}$ , ...,  $L_{c+p}$ , and also  $L_d$ ,  $L_{d+1}$ ,  $L_{d+2}$ , ...,  $L_{d+p}$  are series in geometrical progression, and that  $r \cdot \frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} = m = n.q$ . Since the annuity for  $p$  years on the three lives is equal to  $\frac{L_{b+1}}{L_b} \cdot m + \frac{L_{b+2}}{L_b} \cdot m^2 + \dots + \frac{L_{b+p}}{L_b} \cdot m^p$  it follows

that if  $\frac{L_{b+1}}{L_b} \cdot n + \frac{L_{b+2}}{L_b} \cdot n^2 + \frac{L_{b+3}}{L_b} \cdot n^3 + \dots + \frac{L_{b+p}}{L_b} \cdot n^p = \epsilon \cdot n + \epsilon^2 \cdot n^2 + \epsilon^3 \cdot n^3 + \dots + \epsilon^p \cdot n^p$  that if  $n$  be very nearly equal to  $m$ ,  $\frac{L_{b+1}}{L_b} \cdot n \cdot q + \frac{L_{b+2}}{L_b} \cdot n^2 \cdot q^2 + \text{&c.} + \frac{L_{b+p}}{L_b} \cdot n^p \cdot q^p$  which will be the value of the annuity on the three lives, will be nearly  $= \epsilon \cdot n \cdot q + \epsilon^2 \cdot n^2 \cdot q^2 + \text{&c.} + \epsilon^p \cdot n^p \cdot q^p$ . If  $q$  were equal to unity, or, which is the same thing,  $m = n$ , the equality would be accurate; but it may not be so when  $m$  differs from 1; but the nearer  $n$  is to  $m$ , at least when the difference does not exceed certain limited small quantities, the nearer will be the coincidence. It appears therefore, that if instead of taking the accommodated ratio for  $\epsilon^p$  so that  $\frac{1}{L_b} \times (L_{b+1} + L_{b+2} + L_{b+3} + \dots + L_{b+p}) = \epsilon + \epsilon^2 + \epsilon^3 + \dots + \epsilon^p$  it will be preferable generally to take it so that  $\frac{1}{L_b} \times (n L_{b+1} + n^2 L_{b+2} + n \cdot L_{b+3} \text{ &c.} + \dots + n^p L_{b+p}) = \epsilon + \epsilon^2 + \epsilon^3 + \text{&c.} + \epsilon^p$  in which  $n$  is between  $m$  and 1, the nearer  $m$  the better generally, though possibly not universally so throughout the whole limit. And the second method I use for increasing the accuracy, is to adopt an accommodated ratio, or  $\epsilon^p$ , so that  $\frac{1}{L_b} \times (1,05^{-1} L_{b+1} + 1,05^{-2} L_{b+2} + \text{&c.} + 1,05^{-p} L_{b+p}) = 1,05^{-1} \epsilon + 1,05^{-2} \epsilon^2 + 1,05^{-3} \epsilon^3 + \dots + 1,05^{-p} \epsilon^p$ . Another method which might have its peculiar advantage, is to assume  $\epsilon^p = \frac{L_{b+\frac{1}{2}p}}{L_b}^{\frac{1}{2}p}$  under the idea of using a mean ratio.

### The General Tables.\*

Art. 6. I have had three general tables calculated for fixed periods, Numbers 1, 2, and 3. Number 1, for pe-

\* The chief of the arithmetical operations in the constructions of most of the tables were performed under my direction, by Mr. DAVID JONES, of N<sup>o</sup>. 10, King-street, Soho; and, as far as my leisure would allow, I have endeavoured to assure myself of their accuracy by different inspections.

riods of ten years; that is, for  $\lambda \left( \frac{1-a^{10}}{a^1-1} \right)$ , corresponding to a given value of  $\lambda (a^{10})$ . N<sup>o</sup>. 2, for seven years, or for  $\lambda \left( \frac{1-a^7}{a^1-1} \right)$ , corresponding to  $\lambda (a^7)$ , and the 3d for five years, or for  $\lambda \left( \frac{1-a^5}{a^1-1} \right)$ , corresponding to  $\lambda (a^5)$ ; calculated (whether  $p = 10, 7$  or  $5$ ) for every value of  $\lambda (a^p)$ , answering to  $\bar{3},00$ ;  $\bar{3},01$ ;  $\bar{3},02$ , &c. . . . 0. The first column containing the aforesaid value of  $\lambda (a^p)$ , corresponding to which, in an horizontal line, is placed the log. of  $\frac{1-a^p}{a^1-p}$ , and between each successive value is placed the difference, retaining a decimal figure more; at the head of the other columns for the proportional parts of the differences, are placed a column showing the number of cyphers to be prefixed to the differences entered in the column following, which are headed

$\left\{ \begin{smallmatrix} 1 & 2 & 3 & 4 & 5 \\ 9 & 8 & 7 & 6 & 5 \end{smallmatrix} \right\}$  nearest under 1, 2, 3, 4 and 5, and opposite the number; suppose  $\bar{2},16$  of table log. of  $a^{10}$ , stands

$0275, 0550, 0826, 1101, 1376 \left\{ \right.$ , the upper, with the addition of the 2477, 2202, 1926, 1651, 1376, the upper, with the addition of the two cyphers, give the proportional parts for ,001, 002, 003, ,004, 005: and the under, with the two cyphers, shows the proportional parts for ,009, 008, 007, 006; and the reason of choosing this arrangement, is the advantage which it offers of proof of correctness; thus the sum of the higher and lower numbers of each of the above row with the two cyphers = 002752, which is double ,001376, and equal to the whole difference between the successive terms.

Let it be required to find the logarithm of  $\left( \frac{1-a^{10}}{a^1-1} \right)$ , corresponding to log. of  $a^{10} = 1.7954$ . In the General Table I,

Opposite to 1.79 we have . . . ,88868

For ,005 we have proportional part . 256

For ,0004 . ditto . . . . 20

The sum . ,89144 is the answer.

If log. of  $a^p$  is less than 3,00, then it will be necessary to calculate  $\lambda \left( \frac{1-a^p}{a^{-1}-1} \right)$  by common methods, as the tables do not go lower. And generally it will be then sufficient, omitting  $a^p$ , only to calculate the value of  $-\lambda(a^{-1})$ ; but from this, if more accuracy be required, subtract the number whose common logarithm is  $(1,6378 + \lambda(r)^p)$ .

If  $\lambda \left( \frac{1-a^p}{a^{-1}-1} \right)$  be given, and  $\lambda(a)$  be required, proceed thus,  $\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$  being = ,89144 for example. In Table I, the next value of

$\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$  is ,88868 to which  $\lambda(a^{10})$  corresponding is 1,79

Difference ,00256 belonging to 1.79 gives . . . ,005

Difference ,00020 . . . ditto . . . . . ,0004

$\therefore$  if  $\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right) = \overline{,89144}$  then we have  $\lambda(a^{10}) = \overline{1,7954}$

If  $\lambda(a^p)$  is less than 3, proceed thus: put the given value of  $\lambda \left( \frac{1-a^p}{a^{-1}-1} \right) = \lambda q$ , and we have the common logarithm of  $a = -p \times \lambda(1 + q^{-1}) +$  a small correction if great accuracy be required; which correction is nearly equal to  $p \times$  the number whose common log. is  $\{1.6378 - \lambda q - \overline{p+1} \cdot (1 + q^{-1})\}$

These methods and tables only apply immediately to  $\lambda \left( \frac{1-a^p}{a^{-1}-1} \right)$  when  $a$  is a proper fraction; but if  $a$  be greater than unity, put it equal to  $b^{-1}$ , then will  $b$  be a proper fraction; but  $\frac{1-a^p}{a^{-1}-1} = \frac{a^p-1}{1-a^{-1}} = \frac{b^{-p}-1}{1-b} = b^{-p-1} \frac{1-b^p}{b^{-1}} = a \times \left( \frac{1-b^p}{b^{-1}-1} \right)$ ; conse-

quently  $\lambda \left( \frac{1-a^p}{a^{-1}-1} \right) = \overline{p+1} \cdot \lambda(p) + \lambda \left( \frac{1-b^p}{b^{-1}-1} \right)$  I have likewise had Table IV. calculated, which is a general table, for the common log. of  $\left( \frac{1}{a^{-1}-1} \right)$ , corresponding to a given value of  $\lambda a$ ,

commencing with  $\lambda(a) = 1.7$ ; 1,701; 1,702, &c. with the differences between them. I have not, in this table, had the proportional parts inserted, though it would be attended with advantage, as the table is not meant to be of general use; but only given to be applied for rough purposes, or where accuracy is not particularly required for calculating at once the value of a life annuity for the whole term of life, or the whole remaining terms of life, after a given term, by considering the present value of each successive payment to form the successive terms of a geometrical progression whose first term and common ratio are each equal to  $a$ . And as  $\lambda\left(\frac{1}{a^{\frac{1}{10}}-1}\right)$  will represent the log. of the sum of the said geometrical progression, it will likewise express approximatively the logarithm of the value required. For many purposes, a table of  $\frac{1}{a^{\frac{1}{10}}-1}$ , answering to given values of  $a$ , would be preferable, but not for general purposes.

Art. 7. I have already, in Art. 4 and 5, Chap. II, introduced the term accommodated ratios, or chances, and endeavoured to explain the methods to be adopted to reap the advantage of the ideas there expressed. Table V, for Carlisle, Deparcieux, and Northampton, are the logarithms of tenth terms of the accommodated ratios, or the logarithms of the accommodated chances for living ten years, calculated according to a mode laid down in Art. 5, Chap. II; that is, it expresses for every age, or value of  $b$ , the logarithm of  $\epsilon^{10}$ , when  $\frac{1}{L_b} \times (1,05^{-1}L_{b+1} + 1,05^{-2}L_{b+2} + \&c. \dots 1,05^{-p}L_{b+p})$  is equal to  $1,05^{-1}\epsilon + 1,05^{-2}\epsilon^2 + \&c. \dots 1,05^{-10}\epsilon^{10}$ . and to show, by example, how these are calculated, let it be required to find the logarithm of the accommodated chance for living

ten years, for the age 20, calculated according to the Carlisle table upon the consideration of interest at 5 per cent. Accord-

ing to the Carlisle tables, I find  $\lambda \frac{1}{10} \overbrace{10}^{1,05-1} 20$ ; that is, the logarithm of the annuity of one pound on a life of 20, for ten years, at 5 per cent = .87176, and putting  $a = 105^{\frac{1}{10}} \cdot \beta$ , by hypothesis

we shall have  $\lambda \frac{1}{10} \overbrace{10}^{x} a^x$ ; that is the logarithm of  $(a + a^2 + a^3 \dots a^{10}) = .87176$ ; that is,  $\lambda \left( \frac{1-a^{10}}{a^1-1} \right) = .87176$ ; hence proceeding, as shown above, to find from General Table I.  $\lambda(a^{10})$

$$\text{Having given } .87176 = \lambda \left( \frac{1-a^{10}}{a^1-1} \right)$$

We have next less = .86842 corresponding to . . . . . 1.75

,00334 difference

$$\begin{array}{r} ,00302 \text{ proportional part} \dots \dots \dots ,006 \\ 30 \text{ ditto} \dots \dots \dots ,0006 \\ 2 \text{ ditto} \dots \dots \dots ,00004 \\ \hline \end{array}$$

$$\begin{array}{r} .87176 \text{ corresponds to } \lambda(a^{10}) = 1.75664 \\ \lambda(1,05^{10}) = ,21189 \\ \hline \end{array}$$

1.96853 for the log. of the accommodated chance to live 10 years at the Carlisle mortality.

In the same way may the accommodated chance be found for any other term, when general tables for the term are constructed, and from any other base of interest. I may observe, that by using different rates of interest, as a base for determining the accommodated chances, different degrees of accuracy may be obtained. See Art. 5. Chap. II.

Art. 8. Table VI. is the logarithm of the accommodated chances  $\epsilon$  at every age,  $b$  for living one year, where  $\epsilon$  is of such value that the sum of the geometrical progression  $\frac{\epsilon}{1,05} + \frac{\epsilon^2}{1,05^2} + \text{ &c. ad infinitum}$ , or, which is the same thing,

$\frac{1}{\frac{1,05}{6}-1}$  shall be equal to the value of the whole life annuity at

five per cent. at such age, namely  $\frac{1,05^{-1}}{1} \underline{b}$ ; consequently  $\frac{6}{1,05-1} \times$

$$(1 + \frac{1,05^{-1}}{1} \underline{b}) = \frac{1,05^{-1}}{1} \underline{b}; \therefore \lambda \mathcal{C} = \lambda \left( \frac{1,05^{-1}}{1} \underline{b} \right) + \lambda(1,05) - \lambda \left( \frac{1,05^{-1}}{0} \underline{b} \right).$$

This table is constructed for Carlisle, Deparcieux, and Northampton, and is to be used in conjunction with Table IV., where only a rough value of the contingency is required; and though this table applies as the other tables of accommodated chances, to different rates of interest, still it would be of advantage more particularly *here* for the greater approximation to have similar tables constructed from the

formula  $\lambda(\mathcal{C}) = \lambda \left( \frac{1}{1} \underline{b} \right) + \lambda(r^{-1}) - \lambda \left( \frac{1}{0} \underline{b} \right)$  for different values of  $r$ .

Art. 9. In calculating the value of life annuities for long periods, by means of adding together the values of portions of those periods, the portions of the distant periods contain factors of the real chance of living to these periods, and likewise of the discounted value of the money of which the payment is not immediate; thus if  $t$  be greater than 10,

$$\frac{r}{t} \underline{a, b, c} = \frac{r}{10} \underline{a, b, c} + \frac{r}{10} \underline{a, b, c} = \frac{r}{10} \underline{a, b, c} + \frac{L_{10}: a, b, c}{L_{a, b, c}} \cdot r^{10} \times$$

$\frac{1}{(t-10)} \underline{a+10, b+10, c+10}$ . It will be therefore convenient to have a table of the logarithm of the real chance of living 10, 20, 30 years, &c. and also for other terms; and some of these are given by Tables VII., VIII., IX.

Time will not allow me, for the present, to offer more than a very few examples of the method to be employed in calculating by these tables, which are as follow:

Example 1. Required, according to the Carlisle table, the value of a life annuity, for ten years, on the joint lives 30 and 40, at 3 per cent interest.

In Table VIII. for Carlisle, log. of accommodated

chance for 10 years, at the age 30 . . . .	$= \bar{1}.9552$
Ditto 40 . . . .	$= \bar{1}.9383$
Ditto $\lambda 1,03$ . . . .	$= \bar{1}.8716$
Sum . .	<u><math>\bar{1}.7651 = \lambda (a^{10})</math></u>

In Table I,  $\bar{1}.76$  corresponds to . . . . 8734

In proportional parts, 005 corresponds to .253

Ditto . . . 0001 corresponds to . 5

Consequently  $\bar{1}.7651$  corresponds to . . . . 87604

which is the log. of the required value: the number corresponding to this is 7,5169, for the value of the annuity, according to the Carlisle mortality, at 3 per cent. on the joint lives 30 and 40; and by calculation from Mr. MILNE's tables, I find the value should be 7,5168; the difference of the two is evidently insignificant. In this way I calculated the log. of the value of the life annuity, at the Carlisle mortality, at 3 per cent. for 10 years, for the joint lives 0 and 10, 10 and 20, 20 and 30, 30 and 40, 40 and 50, 50 and 60, to be, 76580; ,90247; ,89139; ,87604; ,86295; ,81067; and the annuity, or the numbers corresponding to the said logarithms,

5,8318; 7,9874; 7,7874; 7,5169; 7,2937; 6,4665;

and, according to calculation from Mr. MILNE's tables, I get

5,8595; 7,992; 7,7906; 7,5168; 7,2916; 6,4679.

The difference between the two sets is insignificant, except

perhaps in the values of  $\frac{1}{10} \left[ \begin{smallmatrix} 1,05 \\ 1 \end{smallmatrix} \right]_{0,10}$ ; that is, the value of the annuity on the joint life of a child just born, with one of the age of 10, at 3 per cent. Had we divided the period in portions, the value might have been obtained as near as we pleased; or we should likewise have obtained greater accuracy, had we assumed an accommodated chance deduced at a more appropriate interest than 5 per cent. See Art. 5, Chap. II.

Example 2. Let it be required to find the value of a life annuity at 3 per cent. for 10 years, at the Carlisle mortality, for the five lives of the age 20, 30, 40, 45 and 50.

In Table VIII. log. of accom. chance for 10 years at age 20 =  $\bar{1.9685}$

Ditto	.	.	.	.	30	=	$\bar{1.9552}$
Ditto	.	.	.	.	40	=	$\bar{1.9383}$
Ditto	.	.	.	.	45	=	$\bar{1.9367}$
Ditto	.	.	.	.	50	=	$\bar{1.9292}$
					$\lambda \frac{1,05}{10}$	=	$\bar{1.8716}$
							$\lambda (a^{10}) = \bar{1.5995}$

This sought in Table I.; thus,  $\bar{1.59}$  giving 79035

,009	427
,0005	23
	<hr/>

gives 7945 the N° to which log. is 6,2352

for the value of  $\frac{1}{10} \left[ \begin{smallmatrix} 1,05 \\ 1 \end{smallmatrix} \right]_{20,30,40,45,50}$ .

Example 3. Let it be required to find the value of  $\frac{1}{10} \left[ \begin{smallmatrix} 1,03 \\ 1 \end{smallmatrix} \right]_{b, b+10}$  Carlisle mortality, when  $b = 10$ , that is, for the whole joint lives of 10 and 20. By dividing the whole in portions of ten

years, the operation will stand thus for  $\frac{1}{10} \left[ \begin{smallmatrix} 1,03 \\ 1 \end{smallmatrix} \right]_{b, b+10}$ .

	$b=10$	$b=20$	$b=30$	$b=40$	$b=50$	$b=60$	$b=70$	$b=80$	
Log. of accom. ratio for 10 years	1.9768	1.9685	1.9552	1.9383	1.9292	1.8318	1.6689	1.3134	from Tab. VIII.
$\lambda(1.03^{-10})$	1.9685	1.9552	1.9383	1.9292	1.8318	1.6689	1.3134	2.6695	Carlisle.
$\lambda(1.03^{-10})$	1.8716	1.8716	1.8716	1.8716	1.8716	1.8716	1.8716	1.8716	
sum . . .	1.8169	1.7953	1.7651	1.7391	1.6326	1.3723	2.8539	3.8545	
No <sup>o</sup> corresponding to sum in Table I.	.90247	.89139	.87604	.86295	.81067	.69156	.48781	.19146	
Log. of ratios for 10 years	1.97438	1.94120	1.89520	1.83292	1.75123	1.57016	1.16886		
$\lambda 1.03^{-10}$ . . .	1.96681	1.92082	1.85854	1.77684	1.59577	1.19448	2.36767		
$\lambda 1.03^{-10}$ . . .	1.87163	1.74325	1.61488	1.48651	1.35814	1.22977	1.10139		
The log. of the present worth of each portion	.70421	.48131	.23157	1.90694	1.39670	2.48222	4.82938		

And the present worth of each, or the numbers corresponding to the last logarithms are arranged below.

For first 10 years	7.9886	As the method by which the logarithms of the present worth of the different portions are found, may not be seen by every reader, I will explain the operation in the third portion; that is, when the logarithm of the portion first found is anticipated for 20 years.
2nd ditto	5.0607	
3d do	3.0291	
4th do	1.7044	Resume . . . . . 87604
5th do	.8071	Table VII. log. of real chance for age . . . . . 1.94120
6th do	.2492	10 living 20 years . . . . .
7th do	.0303	Ditto 20 years living . . . . . 1.92082
8th do	.0007	$\lambda(1.03^{-20})$ . . . . . 1.74325
sum	18.8701	.48131

which differs but insignificantly from Mr. MILNE's table, which gives 18.873. In a similar way, I find the value of the joint lives for ages 20 and 30, at 3 per cent. and Carlisle mortality to be 16.745; which, according to Mr. MILNE's table, should be 16.749; which appears to be an insignificant difference.

Example 4. To find, when particular accuracy is not required, according to the formula for the whole of life,

the approximate value of  $\lambda \frac{1}{1} \left[ a, a+10 \right]^{1.03^{-1}}$  at the Carlisle mortality, when  $a = 10, 20, 30, \&c.$  call the logarithm of accommodated ratios for an unlimited time at the age  $a$ ,  $R_a$  standing for the accommodated ratio in Table VI. at the age  $a$ .

$a =$	10	20	30	40	50	60	70	80	90
$R_a$	1.99529	1.99455	1.99265	1.98991	1.98546	1.97514	1.95755	1.92461	1.86660
$R_{a+10}$	1.99455	1.99265	1.98991	1.98546	1.97514	1.95755	1.92461	1.86660	1.81282
$\lambda_{1,03}^{-1}$	1.98716	1.98716	1.98716	1.98716	1.98716	1.98716	1.98716	1.98716	1.98716
	1.97700	1.97436	1.96972	1.96253	1.94776	1.91985	1.86932	1.77837	1.66658
Log. which corresponds to	1.26451 {	1.20975 .00631	1.13083 .01062	1.03886 .00641	.88674 .00667	.68817 .00502	.45337 .00123	.17571 .00093	
		1.21606	1.14145	1.04527	.89341	.69319	.45460	.17664	
Numbers . .	18.387	16.446	13.850	11.099	7.824	4.9339	2.8485	1.5019	
Instead of . .	18.373	16.749	14.449	11.954	8.729	5.565	3.229	1.589	

To find the value corresponding to 1.66658, not in the table, find the number corresponding complement of the log. 1.66658, which number is 2,159; subtract 1, and find the complement of the log. which is = 1.9359165, whose number is ,8628. Mr. MILNE's table gives .979. But as it is not always the same rate of interest which gives the best accommodated ratios, in order to try when, for instance, the interest of money is 3 per cent. what rate of interest should be used in determining the ratios, use the following table:\*

Interest.

1.08	$\lambda (1.08^{-1} \times 1.03) = 1.979$
1.07	$\lambda (1.07^{-1} \times 1.03) = 1.983$
1.06	$\lambda (1.06^{-1} \times 1.03) = 1.987$
1.05	$\lambda (1.05^{-1} \times 1.03) = 1.991$
1.04	$\lambda (1.04^{-1} \times 1.03) = 1.996$

nearly;

\* This is not given as a perfect and unerring rule, but as a method in many cases useful, and which would be perfect for the accommodated ratio of one of the lives, if the other lives followed an exact geometrical ratio throughout; and that the real geometrical ratios were in that case used for them, provided that instead of comparing the said sum with the small table, we take for the base of interest the number whose logarithm is —  $\lambda(1,03)$ , when the interest is 3 per cent.; and it is to be recollect that the methods is only given as a rough approximation.

Add the logarithm of accommodated ratios, as given in the Table VI. of all the lives but one in question, together, and see which of those rates of interest it nearest agrees with, and use that to calculate the life left, and proceed so for

every life ; thus for  $\frac{1}{1} \left[ \begin{smallmatrix} 30, 40 \\ 40 \end{smallmatrix} \right]$  ; to find the rate of interest for 30, I observe that  $R = \frac{1}{40} 1.9899$  agrees nearest with 6 per cent. in the little table, and  $R = \frac{30}{40} 1.99265$  agrees nearest with 5 per cent., I therefore take 6 per cent. for the age 30, and for the other I take 5 per cent. : proceed thus :

### Example 5.

R if calculated at 6 per cent.	$\frac{30}{30} 1.99316$
R per table	$\frac{40}{40} 1.98991$
$\lambda 1.05^{-1}$	$= \frac{1.98716}{1.97023}$
Proportionate parts	$\frac{1.14558}{.00327}$
To which logarithm	$\frac{1.14885}{.00102}$
The N° corresponding is	$\frac{14.088}{1.06438}$
Instead of	$\frac{14.449}{11.598}$

### Example 6.

$\frac{1}{1} \left[ \begin{smallmatrix} 40, 50 \\ 50 \end{smallmatrix} \right]$	$\frac{1.03^{-1}}{1.96408}$
R at 6 per cent.	$\frac{40}{40} 1.99060$
R at 6 per cent.	$\frac{50}{50} 1.98632$
	$\frac{1.98716}{1.96408}$
	$\frac{1.05336}{.00102}$
	$\frac{1.06438}{1.05336}$
Instead of	$\frac{11.598}{11.954}$

### Example 7.

$\frac{1}{1} \left[ \begin{smallmatrix} 50, 60 \\ 60 \end{smallmatrix} \right]$	$\frac{1.03^{-1}}{1.95074}$
R at 8 per cent.	$\frac{50}{50} 1.98759$
R at 6 per cent.	$\frac{60}{60} 1.97599$
$\lambda 1.03^{-1}$	$= \frac{1.98716}{1.95074}$
	$\frac{.91357}{.00687}$
	$\frac{.92044}{.8318}$
which log. corresponds	$\frac{.8318}{.8729}$
instead of	

I observe that I have not given any table of the logarithm of the accommodated ratios for an unlimited term, except that calculated with 5 per cent. as a radix ; but by the assistance of a table of life annuities, for single life at different rates per cent., this will enable us, independent of certain exceptions, to derive the quantity for the same rates per cent. for any radix at the per cent. contained in the second table ; thus to find R Carlisle mortality, radix 8 per cent. I look to the Carlisle table of single lives at 8 per cent., and I find the value of the annuity on the life of  $50 = 8.987$ , I search the age to which this will correspond at 5 per cent. and I find sufficiently nearly 59.82 for the age corresponding, to which from my table (with the radix at 5 per cent.) for the log. of ratios I find 1.97536 ; to this I add log. of  $\frac{1.08}{1.05}$  ; that is, ,01223, and we get 1.98759, the same as given on the other side. This method is accurately consistent with the definition of accommodated ratios for unlimited periods ; and if this description of accommodated ratios at a certain rate per cent. be given for one table, for which at the same rate per cent. we have the value of single lives, we may find the same description of accommodated ratios for any other table of mortality for which, at the same rate per cent. we have a table of the value of single lives : thus, suppose the logarithm of this description of accommodated ratios be given for the Carlisle table at five per cent., and the same be required for the Northampton for the age 60, at the same rate ;  $\frac{1}{1} \overline{60}^{-1}$  Northampton = 8,392, this being sought in the Carlisle

table for  $\frac{1}{1} \left[ x \right]^{-1}$  gives  $x = 62,41$  for the corresponding age; seek the logarithm of accommodated ratios for an unlimited term, corresponding to this for Carlisle, for the age 6,241, and we have  $\bar{1.9723}$ , agreeing with the table given.

Previously to concluding this chapter, I shall add a small table, which will be found very useful in the application of the methods here proposed.

$n$	Log. of $1,03^{-n}$	Log. of $1,035^{-n}$	Log. of $1,04^{-n}$	Log. of $1,045^{-n}$	Log. of $1,05^{-n}$
1	1.9871628	1.9850597	1.9829667	1.9808837	1.9788107
2	1.9743256	1.9701193	1.9659333	1.9617674	1.9576214
3	1.9614883	1.9551790	1.9489000	1.9426511	1.9364321
4	1.9486511	1.9402386	1.9318666	1.9235348	1.9152428
5	1.9358139	1.9252983	1.9148333	1.9044185	1.8940535
6	1.9229767	1.9103579	1.8978000	1.8853023	1.8728642
7	1.9101394	1.8954176	1.8807666	1.8661860	1.8516749
8	1.8973022	1.8804772	1.8637333	1.8470697	1.8304856
9	1.8844650	1.8655369	1.8466999	1.8279534	1.8092963
10	1.8716278	1.8505965	1.8296666	1.8088371	1.7881070

General Table I.  $\lambda(a^{10})$ ,  $\lambda\left(\frac{1-a^{10}}{a-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5
3.00	,00163 ,00199,6	,00	0200	0399	0599	0798	0998	3.25	,05295 ,00211,7	,00	0212	0423	0635	0847	1059
3.01	,0036,2 ,00200,3	0200	0401	0601	0801	1002		3.26	,05506 ,00212,2	0212	0424	0637	0849	1061	
3.02	,00563 ,00200,6	0201	0401	0602	0802	1003		3.27	,05719 ,00212,7	0213	0425	0638	0851	1064	
3.03	,00763 ,00201,0	0201	0402	0603	0804	1005		3.28	,05931 ,00213,2	0213	0426	0640	0853	1066	
3.04	,00964 ,00201,5	0202	0403	0605	0806	1008		3.29	,06144 ,00213,7	0214	0427	0641	0855	1069	
3.05	,01166 ,00202,0	0202	0404	0606	0808	1010		3.30	,06358 ,00214,3	0214	0429	0643	0857	1072	
3.06	,01368 ,00202,4	0202	0405	0607	0810	1012		3.31	,06572 ,00214,8	0215	0430	0644	0859	1074	
3.07	,01570 ,00202,8	0203	0406	0608	0811	1014		3.32	,06787 ,00215,3	0215	0431	0646	0861	1077	
3.08	,01773 ,00203,4	0203	0407	0610	0814	1017		3.33	,07002 ,00216,0	0216	0432	0648	0864	1080	
3.09	,01976 ,00203,8	0204	0408	0611	0815	1019		3.34	,07218 ,00216,4	0216	0433	0649	0866	1082	
3.10	,02180 ,00204,3	0204	0409	0613	0817	1022		3.35	,07435 ,00217,0	0217	0434	0651	0868	1085	
3.11	,02384 ,00204,7	0205	0409	0614	0819	1024		3.36	,07652 ,00217,5	0218	0435	0653	0870	1089	
3.12	,02589 ,00205,2	0205	0410	0616	0821	1026		3.37	,07869 ,00218,0	0218	0436	0654	0872	1090	
3.13	,02794 ,00205,7	0206	0411	0617	0823	1029		3.38	,08087 ,00218,7	0219	0437	0656	0875	1094	
3.14	,03000 ,00206,1	0206	0412	0618	0824	1031		3.39	,08306 ,00219,2	0219	0438	0658	0877	1096	
3.15	,03206 ,00206,5	0207	0413	0620	0826	1033		3.40	,08525 ,00219,7	0220	0439	0659	0879	1099	
3.16	,00207 ,00207,3	0207	0415	0622	0829	1037		3.41	,08745 ,00220,4	0220	0441	0661	0882	1102	
3.17	,03620 ,00207,6	0208	0415	0623	0830	1038		3.42	,08965 ,00221,0	0221	0442	0663	0884	1105	
3.18	,03827 ,00208,1	0208	0416	0624	0832	1041		3.43	,09186 ,00221,4	0221	0443	0664	0886	1107	
3.19	,04036 ,00208,6	0209	0417	0626	0834	1043		3.44	,09408 ,00222,1	0222	0444	0666	0888	1111	
3.20	,04244 ,00209,1	0209	0418	0627	0836	1046		3.45	,09630 ,00222,6	0223	0445	0668	0890	1113	
3.21	,04453 ,00209,6	0210	0419	0629	0838	1048		3.46	,09852 ,00223,2	0223	0446	0670	0893	1116	
3.22	,04663 ,00210,1	0210	0420	0630	0840	1051		3.47	,10076 ,00223,8	2009	1786	1562	1339		
3.23	,04873 ,00210,6	0211	0421	0632	0842	1053		3.48	,10300 ,00224,4	2014	1790	1567	1343		
3.24	,05084 ,00211,1	0211	0422	0633	0844	1056		3.49	,10524 ,00225,0	2025	1795	1571	1346		
		1900	1689	1478	1267					2025	1800	1575	1350		

General Table I.  $\lambda(a^{10})$ ,  $\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
3.50	,10749 ,00225,6	,000	0226	0451	0677	0902	1128	3.75	,16581 ,00242,0	,000	0242	0484	0726	0968	1210
3.51	,10975 ,00226,2	2030	1805	1579	1354	1131		3.76	,16823 ,00242,7	0243	0485	0728	0971	1214	
3.52	,11201 ,00226,8	0226	0452	0679	0905	1134		3.77	,17065 ,00243,5	0244	0487	0731	0974	1218	
3.53	,11428 ,00227,5	2041	1814	1588	1361			3.78	,17309 ,00244,2	0244	0488	0733	0977	1221	
3.54	,11655 ,00228,1	0228	0455	0683	0910	1138		3.79	,17553 ,00244,9	0245	0490	0735	0980	1225	
3.55	,11883 ,00228,7	2058	1830	1601	1372			3.80	,17798 ,00245,6	0246	0491	0737	0982	1228	
3.56	,12112 ,00229,2	0229	0458	0688	0917	1146		3.81	,18044 ,00246,4	0246	0493	0739	0986	1232	
3.57	,12341 ,00230,1	2063	1834	1604	1375			3.82	,18290 ,00247,1	0247	0494	0741	0988	1236	
3.58	,12571 ,00230,6	0230	0460	0690	0920	1151		3.83	,18537 ,00247,8	0224	1977	1730	1482		
3.59	,12802 ,00231,2	2071	1841	1611	1381			3.84	,18785 ,00248,6	0230	1982	1735	1487		
3.60	,13033 ,00231,9	0232	0464	0696	0928	1160		3.85	,19034 ,00249,3	0249	0499	0748	0997	1247	
3.61	,13265 ,00232,5	2087	1855	1623	1391			3.86	,19284 ,00250,1	0244	1994	1745	1496		
3.62	,13497 ,00233,0	0233	0465	0698	0930	1163		3.87	,19533 ,00250,9	0250	0500	0750	1000	1251	
3.63	,13730 ,00233,8	2093	1860	1628	1395			3.88	,19784 ,00251,6	0251	0502	0753	1004	1255	
3.64	,13964 ,00234,5	0234	0466	0699	0932	1165		3.89	,20035 ,00252,4	0258	0503	0755	1006	1258	
3.65	,14199 ,00235,1	2104	1870	1637	1403			3.90	,20288 ,00253,2	0252	0505	0757	1010	1262	
3.66	,14434 ,00235,8	0235	0468	0701	0935	1169		3.91	,20541 ,00254,1	0253	0506	0760	1013	1266	
3.67	,14670 ,00236,6	2111	1876	1642	1407			3.92	,20795 ,00254,7	0254	0508	0762	1016	1271	
3.68	,14906 ,00237,1	0237	0473	0710	0940	1183		3.93	,21050 ,00255,7	0287	2033	1779	1525		
3.69	,15143 ,00237,9	2129	1893	1656	1420			3.94	,21306 ,00256,3	0255	0509	0764	1019	1274	
3.70	,15381 ,00238,5	0239	0477	0716	0954	1193		3.95	,21562 ,00257,2	0256	0514	0772	1029	1286	
3.71	,15620 ,00239,2	2147	1908	1670	1431			3.96	,21819 ,00258,0	0258	0515	0780	1043		
3.72	,15859 ,00239,9	2153	1914	1674	1435			3.97	,22077 ,00258,8	0259	0516	0774	1032	1290	
3.73	,16099 ,00240,6	0240	0480	0720	0960	1200		3.98	,22336 ,00259,6	0260	0518	0776	1035	1294	
3.74	,16339 ,00241,3	2159	1919	1679	1439			3.99	,22559 ,00260,4	0260	0519	0779	1038	1298	
		0241	0481	0722	0962	1203				0260	0521	0781	1042	1302	
		2165	1925	1684	1444					0260	0521	0781	1042	1302	
		0241	0483	0724	0965	1207				0264	0523	0783	1043	1302	
		2172	1930	1689	1448					0264	0523	0783	1043	1302	

General Table I.  $\lambda (a^{10})$ ,  $\lambda \left( \frac{1-a^{10}}{a-1} \right)$ .

$\lambda (a^{10})$	$\lambda \left( \frac{1-a^{10}}{a-1} \right)$		1 9	2 8	3 7	4 6	5	$\lambda (a^{10})$	$\lambda \left( \frac{1-a^{10}}{a-1} \right)$		1 9	2 8	3 7	4 6	5
2.00	.22856 ,00261,4	,00	0261	0523	0784	1046	1307	2.25	,29652 ,00283,9	,00	0284	0568	0852	1136	1420
2.01	.23117 ,00262,0	0262	0524	0786	1048	1310	2.26	,29936 ,00284,9	0285	0570	0855	1140	1425		
2.02	.23379 ,00263,0	0263	0526	0789	1052	1315	2.27	,30221 ,00285,9	0286	0572	0858	1144	1430		
2.03	.23642 ,00263,8	0264	0528	0791	1055	1319	2.28	,30507 ,00286,8	0287	0574	0860	1147	1434		
2.04	.23906 ,00264,7	0265	0529	0794	1059	1324	2.29	,30794 ,00287,9	0288	0576	0864	1152	1440		
2.05	.24171 ,00265,6	0266	0531	0797	1062	1328	2.30	,31081 ,00288,9	0289	0578	0867	1156	1445		
2.06	.24436 ,00266,3	0266	0533	0799	1065	1332	2.31	,31370 ,00289,9	0290	0580	0870	1160	1450		
2.07	.24703 ,00267,0	0267	0534	0801	1068	1335	2.32	,31660 ,00290,9	0291	0582	0873	1164	1455		
2.08	.24970 ,00268,5	0269	0537	0806	1074	1343	2.33	,31951 ,00291,9	0292	0584	0876	1168	1460		
2.09	.25238 ,00269,0	0269	0538	0807	1076	1345	2.34	,32243 ,00293,0	0293	0586	0879	1172	1465		
2.10	.25507 ,00269,9	0270	0540	0810	1080	1350	2.35	,32536 ,00294,0	0294	0588	0882	1176	1470		
2.11	.25777 ,00270,8	0271	0542	0812	1083	1354	2.36	,32830 ,00295,0	0295	0590	0885	1180	1475		
2.12	.26048 ,00271,7	0272	0543	0815	1087	1359	2.37	,33125 ,00296,1	0296	0592	0888	1184	1481		
2.13	.26320 ,00272,6	0273	0545	0818	1090	1363	2.38	,33421 ,00297,1	0297	0594	0891	1188	1485		
2.14	.26592 ,00273,5	0274	0547	0821	1094	1368	2.39	,33718 ,00298,2	0298	0596	0895	1193	1491		
2.15	.26866 ,00274,6	0275	0549	0824	1098	1373	2.40	,34016 ,00299,3	0299	0599	0898	1197	1497		
2.16	.27140 ,00275,2	0275	0550	0826	1101	1376	2.41	,34316 ,00300,3	0300	0601	0901	1201	1502		
2.17	.27415 ,00276,3	0276	0553	0829	1105	1382	2.42	,34316 ,00301,4	0301	0603	0904	1206	1507		
2.18	.27692 ,00277,2	0277	0554	0832	1109	1386	2.43	,34917 ,00302,5	0303	0605	0908	1210	1513		
2.19	.27969 ,00278,1	0278	0556	0834	1112	1391	2.44	,35220 ,00303,6	0304	0607	0911	1214	1518		
2.20	.28247 ,00279,1	0279	0558	0837	1116	1396	2.45	,35523 ,00304,7	0305	0609	0914	1219	1524		
2.21	.28526 ,00280,1	0280	0560	0840	1120	1401	2.46	,35828 ,00305,8	0306	0612	0917	1223	1529		
2.22	.28806 ,00281,1	0281	0562	0843	1124	1406	2.47	,36134 ,00306,9	0307	0614	0921	1228	1535		
2.23	.29087 ,00281,9	0282	0564	0846	1128	1410	2.48	,36441 ,00308,1	0308	0616	0924	1232	1541		
2.24	.29369 ,00282,9	0283	0566	0849	1132	1415	2.49	,36749 ,00309,2	0309	0618	0928	1237	1546		
		2546	2263	1980	1697				2783	2474	2164	1855			

General Table I.  $\lambda(a^{10})$ ,  $\lambda\left(\frac{1-a^{10}}{a-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5
2.50	,37058 ,00310,3	,00	0310 2794	0621 2482	0931 2172	1241 1862	1552	2.75	,45174 ,00340,8	,00	0341 3067	0682 2726	1022 2386	1363 2045	1704
2.51	,37368 ,00311,5		0312 2804	0623 2492	0935 2181	1246 1869	1558	2.76	,45515 ,00342,3		0342 3081	0685 2738	1027 2396	1369 2054	1712
2.52	,37680 ,00312,6		0313 2813	0625 2501	0938 2188	1250 1876	1563	2.77	,45857 ,00343,5		0343 3092	0687 2748	1031 2405	1374 2061	1718
2.53	,37993 ,00313,8		0314 2824	0628 2510	0941 2197	1255 1883	1569	2.78	,46201 ,00345,0		0345 3105	0690 2760	1035 2415	1380 2070	1725
2.54	,38306 ,00314,9		0315 2834	0630 2519	0945 2204	1260 1889	1575	2.79	,46546 ,00346,1		0346 3115	0692 2769	1038 2423	1384 2077	1731
2.55	,38621 ,00316,1		0316 2845	0632 2529	0948 2213	1265 1897	1581	2.80	,46892 ,00347,5		0348 3128	0695 2780	1043 2433	1390 2085	1738
2.56	,38937 ,00317,3		0317 2856	0635 2538	0952 2221	1269 1904	1587	2.81	,47239 ,00348,9		0349 3140	0698 2791	1047 2442	1396 2093	1745
2.57	,39255 ,00318,5		0319 2867	0637 2548	0956 2230	1274 1911	1593	2.82	,47588 ,00350,2		0350 3152	0700 2802	1051 2451	1410 2101	1751
2.58	,39573 ,00319,6		0320 2876	0639 2557	0959 2237	1278 1918	1598	2.83	,47938 ,00351,6		0352 3164	0703 2813	1055 2461	1406 2110	1758
2.59	,39893 ,00320,8		0321 2887	0642 2566	0962 2246	1283 1925	1604	2.84	,48290 ,00353,0		0353 3177	0706 2824	1059 2471	1412 2118	1765
2.60	,40213 ,00322,0		0322 2898	0644 2576	0966 2254	1288 1932	1610	2.85	,48643 ,00354,3		0354 3189	0709 2834	1063 2486	1417 2126	1772
2.61	,40536 ,00323,3		0323 2910	0647 2586	0970 2263	1293 1940	1617	2.86	,48997 ,00355,8		0356 3202	0712 2846	1067 2491	1423 2135	1779
2.62	,40859 ,00324,5		0325 2921	0649 2596	0974 2272	1298 1947	1623	2.87	,49353 ,00357,1		0357 3214	0714 2857	1071 2500	1428 2143	1786
2.63	,41183 ,00325,6		0326 2930	0651 2605	0977 2279	1302 1954	1628	2.88	,49710 ,00358,5		0359 3227	0717 2868	1076 2510	1434 2151	1793
2.64	,41509 ,00326,8		0327 2941	0654 2614	0980 2288	1307 1961	1634	2.89	,50069 ,00360,0		0360 3240	0720 2880	1080 2520	1440 2160	1800
2.65	,41836 ,00328,3		0328 2955	0657 2626	0985 2298	1313 1970	1642	2.90	,50429 ,00361,4		0361 3253	0723 2891	1084 2530	1446 2168	1807
2.66	,42164 ,00329,4		0329 2965	0659 2635	0988 2306	1318 1976	1647	2.91	,50790 ,00362,7		0363 3264	0725 2902	1088 2539	1451 2176	1814
2.67	,42493 ,00330,6		0331 2975	0661 2645	0992 2314	1322 1984	1653	2.92	,51153 ,00364,2		0364 3278	0728 2914	1093 2549	1457 2185	1821
2.68	,42833 ,00331,9		0332 2987	0664 2655	0996 2323	1328 1991	1660	2.93	,51517 ,00365,6		0366 3290	0731 2925	1097 2559	1462 2194	1828
2.69	,43156 ,00333,2		0333 2999	0666 2666	1000 2332	1333 1999	1666	2.94	,51883 ,00367,1		0367 3304	0734 2937	1101 2570	1468 2203	1836
2.70	,43490 ,00334,4		0334 3010	0669 2675	1003 2341	1338 2006	1672	2.95	,52250 ,00368,5		0369 3317	0737 2948	1106 2580	1474 2211	1843
2.71	,43824 ,00335,7		0336 3021	0671 2686	1007 2350	1343 2014	1679	2.96	,52618 ,00369,9		0370 3329	0740 2959	1110 2589	1480 2219	1850
2.72	,44159 ,00337,1		0337 3034	0674 2697	1011 2360	1348 2023	1686	2.97	,52988 ,00371,4		0371 3343	0743 2971	1114 2600	1486 2228	1857
2.73	,44496 ,00338,2		0338 3044	0676 2706	1015 2367	1353 2029	1691	2.98	,53360 ,00372,9		0373 3356	0746 2983	1119 2610	1492 2237	1865
2.74	,44835 ,00339,6		0340 3056	0679 2717	1019 2377	1358 2038	1698	2.99	,53732 ,00374,4		0374 3370	0749 2995	1123 2620	1498 2246	1872

General Table I.  $\lambda (a^{10})$ ,  $\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$ .

$\lambda (a^{10})$	$\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$		1 9	2 8	3 7	4 6	5	$\lambda (a^{10})$	$\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$		1 9	2 8	3 7	4 6	5
1.00	54107 ,00375,8	,00	0376	0752	1127	1503	1879	1.25	,63964 ,00415,3	,00	0415	0831	1246	1661	2077
1.01	54483 ,00377,4	0377	0755	1132	1510	1887		1.26	,64380 ,00416,7	0417	0833	1250	1667	2084	
1.02	54860 ,00378,7	0379	0757	1136	1515	1894		1.27	,64796 ,00418,4	0418	0837	1255	1674	2092	
1.03	55239 ,00380,3	0380	0761	1141	1521	1902		1.28	,65215 ,00420,0	0420	0840	1260	1680	2100	
1.04	55619 ,00381,9	0382	0764	1146	1528	1910		1.29	,65635 ,00421,7	0422	0843	1265	1687	2109	
1.05	56001 ,00383,3	0383	0767	1150	1533	1917		1.30	,66056 ,00423,3	0423	0847	1270	1693	2117	
1.06	56384 ,00384,9	0385	0770	1155	1540	1925		1.31	,66480 ,00425,0	0425	0850	1275	1700	2125	
1.07	56769 ,00386,4	0386	0773	1159	1546	1932		1.32	,66905 ,00426,7	0427	0853	1280	1707	2134	
1.08	57156 ,00388,0	0388	0776	1164	1552	1940		1.33	,67331 ,00428,4	0428	0857	1285	1714	2143	
1.09	57544 ,00389,4	0389	0779	1168	1558	1947		1.34	,67760 ,00430,1	0430	0860	1290	1720	2151	
1.10	57933 ,00391,0	0391	0782	1173	1564	1955		1.35	,68190 ,00431,7	0432	0863	1295	1727	2159	
1.11	58324 ,00392,5	0393	0785	1178	1570	1963		1.36	,68622 ,00434,0	0434	0868	1302	1736	2170	
1.12	58717 ,00394,2	0394	0788	1183	1577	1971		1.37	,69056 ,00434,7	0435	0869	1304	1739	2174	
1.13	59111 ,00395,9	0396	0792	1188	1584	1980		1.38	,69490 ,00437,0	0437	0874	1311	1748	2185	
1.14	59507 ,00397,1	0397	0794	1191	1588	1986		1.39	,69927 ,00438,7	0439	0877	1316	1755	2194	
1.15	59904 ,00398,8	0399	0798	1196	1595	1994		1.40	,70366 ,00440,4	0440	0881	1321	1762	2202	
1.16	60303 ,00400,5	0401	0801	1202	1602	2003		1.41	,70806 ,00442,1	0442	0884	1326	1768	2211	
1.17	60703 ,00402,1	0402	0804	1206	1608	2011		1.42	,71249 ,00443,8	0444	0888	1331	1775	2219	
1.18	61105 ,00403,7	0404	0827	1211	1615	2019		1.43	,71692 ,00445,0	0445	0890	1335	1780	2225	
1.19	61509 ,00405,3	0405	0811	1216	1621	2027		1.44	,72137 ,00448,0	0448	0896	1344	1792	2240	
1.20	61914 ,00406,9	0407	0814	1221	1628	2035		1.45	,72585 ,00449,1	0449	0898	1347	1796	2246	
1.21	62321 ,00408,5	0409	0817	1226	1634	2043		1.46	,73035 ,00450,8	0451	0902	1352	1803	2254	
1.22	62729 ,00410,1	0410	0820	1230	1640	2051		1.47	,73485 ,00452,7	0453	0905	1358	1811	2264	
1.23	63140 ,00411,8	0412	0824	1235	1647	2059		1.48	,73938 ,00454,4	0454	0909	1363	1818	2272	
1.24	63551 ,00413,1	0413	0826	1239	1652	2066		1.49	,74393 ,00456,1	0456	0912	1368	1824	2281	
		3718	3305	2892	2489					4105	3649	3193	2737		

General Table I.  $\lambda(a^{10})$ ,  $\lambda\left(\frac{1-a^{10}}{a-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5	
1.50	,74849 ,00458,0	,00	0458	0916	1374	1832	2290	1.75	,86842 ,00503,7	,00	0504	1007	1511	2015	2519	
1.51	,75307 ,00459,8	0460	0920	1379	1839	2299		1.76	,87346 ,00505,6	0506	4533	4030	3526	3022	2528	
1.52	,75766 ,00461,6	0462	0923	1385	1846	2308		1.77	,87851 ,00507,4	0507	4550	4045	3539	3034	2537	
1.53	,76228 ,00463,3	0463	0927	1390	1853	2317		1.78	,88359 ,00509,4	0509	4567	4059	3552	3044	2547	
1.54	,76691 ,00465,2	0465	0930	1396	1861	2326		1.79	,88868 ,00511,2	0511	4585	4075	3566	3056	2556	
1.55	,77156 ,00466,7	0467	0933	1400	1867	2334		1.80	,89379 ,00513,1		0513	4126	1539	2052	2566	
1.56	,77623 ,00469,0	0469	0938	1407	1876	2345		1.81	,89892 ,00515,0	0515	4618	4105	3592	3079		
1.57	,78092 ,00470,4	0470	0941	1411	1882	2352		1.82	,90407 ,00516,8	0517	4635	4120	3605	3090	2575	
1.58	,78562 ,00472,4	0472	0945	1417	1890	2362		1.83	,90924 ,00518,8	0519	4651	4134	3618	3101		
1.59	,79035 ,00474,1	0474	0948	1422	1896	2371		1.84	,91443 ,00520,6	0521	4669	4150	3632	3113		
1.60	,79509 ,00476,0	0476	0952	1428	1904	2380		1.85	,91964 ,00522,6	0523	4703	4181	3658	3136		
1.61	,79985 ,00477,8	0478	0956	1433	1911	2389		1.86	,92486 ,00524,4	0524	4720	4195	3671	3146		
1.62	,80463 ,00479,6	0480	0959	1439	1918	2398		1.87	,93011 ,00526,1	0526	4735	4209	3683	3157		
1.63	,80942 ,00481,5	0482	0963	1445	1926	2408		1.88	,93537 ,00528,3	0528	4755	4226	3698	3170		
1.64	,81424 ,00483,3	0483	0967	1450	1933	2417		1.89	,94065 ,00530,0	0530	4770	4240	3710	3180		
1.65	,81907 ,00485,7	0486	0971	1457	1943	2429		1.90	,94595 ,00532,0		0532	4788	4256	3724	3192	
1.66	,82393 ,00486,4	0486	0973	1459	1946	2432		1.91	,95127 ,00533,8	0534	4804	4270	3737	3203		
1.67	,82879 ,00488,8	0489	0978	1466	1955	2444		1.92	,95661 ,00535,8	0536	4822	4286	3751	3215		
1.68	,83368 ,00491,7	0492	0983	1475	1967	2459		1.93	,96197 ,00537,6	0538	4838	4301	3763	3226		
1.69	,83859 ,00492,5	0493	0985	1478	1970	2463		1.94	,96734 ,00539,6	0540	4846	4317	3777	3238		
1.70	,84351 ,00494,4	0494	0989	1483	1978	2472		1.95	,97274 ,00541,4		0541	4873	4331	3790	3248	
1.71	,84846 ,00496,3	0496	0993	1489	1985	2482		1.96	,97815 ,00543,3	0543	4890	4346	3703	3260		
1.72	,85342 ,00498,1	0498	0996	1494	1992	2491		1.97	,98359 ,00545,2	0545	4907	4362	3716	3271		
1.73	,85840 ,00499,9	0500	1000	1500	2000	2500		1.98	,98904 ,00547,2	0547	4925	4378	3830	3283		
1.74	,86340 ,00501,9	0502	1004	1506	2008	2510		1.99	,99451							

General Table II.  $\lambda(a^t), \lambda\left(\frac{1-a^t}{a^{-1}-1}\right)$ .

$\lambda(a^t)$	$\lambda\left(\frac{1-a^t}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^t)$	$\lambda\left(\frac{1-a^t}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
3.00	1,77356 ,00227,0	,00	0227	0454	0681	0908	1135	3.25	1,83164 ,00238,5	,00	0239	0477	0716	0954	1193
	2043			1816	1589	1362			1,83403 ,00239,0		2147	1908	1670	1431	
3.01	1,77583 ,00227,4		0227	0455	0682	0910	1137	3.26	1,83641 ,00239,5		0239	0478	0717	0956	1195
	2047			1819	1592	1364			1,83881 ,00240,0		2151	1912	1673	1434	
3.02	1,77810 ,00227,8		0228	0456	0683	0911	1139	3.27	1,84121 ,00240,5		0240	0479	0719	0958	1198
	2050			1822	1595	1367			1,84421 ,00240,5		2156	1916	1677	1437	
3.03	1,78038 ,00228,3		0228	0457	0685	0913	1142	3.28	1,84602 ,00241,5		0240	0480	0720	0960	1200
	2055			1826	1598	1370			1,85086 ,00242,5		2160	1920	1680	1440	
3.04	1,78266 ,00228,6		0229	0457	0686	0914	1143	3.29	1,85328 ,00243,2		0241	0481	0722	0962	1203
	2057			1829	1600	1372			1,85571 ,00243,6		2165	1924	1684	1443	
3.05	1,78495 ,00229,2		0229	0458	0688	0917	1146	3.30	1,85815 ,00244,1		0241	0482	0723	0964	1205
	2063			1834	1604	1375			1,86059 ,00244,7		2168	1927	1686	1445	
3.06	1,78724 ,00229,6		0230	0460	0689	0918	1148	3.31	1,86304 ,00245,2		0242	0483	0725	0966	1208
	2066			1837	1607	1378			1,86538 ,00245,8		2174	1932	1691	1449	
3.07	1,78954 ,00230,0		0230	0460	0690	0920	1150	3.32	1,86844 ,00246,2		0242	0484	0726	0968	1210
	2070			1840	1610	1380			1,87086 ,00246,9		2177	1935	1693	1451	
3.08	1,79184 ,00230,5		0231	0461	0692	0922	1153	3.33	1,87328 ,00247,5		0243	0485	0728	0970	1213
	2075			1844	1614	1383			1,87554 ,00247,9		2183	1940	1698	1455	
3.09	1,79414 ,00230,8		0231	0462	0692	0923	1154	3.34	1,87880 ,00248,5		0243	0486	0730	0973	1216
	2077			1846	1616	1385			1,88032 ,00248,5		2189	1946	1702	1459	
3.10	1,79645 ,00231,4		0231	0463	0694	0926	1157	3.35	1,88364 ,00249,1		0244	0487	0731	0974	1218
	2083			1851	1620	1388			1,88602 ,00249,7		2192	1949	1705	1462	
3.11	1,79877 ,00231,8		0232	0464	0695	0927	1159	3.36	1,88841 ,00249,1		0244	0488	0732	0976	1221
	2086			1854	1623	1391			1,89079 ,00249,7		2197	1953	1709	1463	
3.12	1,80108 ,00232,2		0232	0464	0697	0929	1161	3.37	1,89317 ,00249,7		0245	0489	0734	0979	1224
	2090			1858	1625	1393			1,89555 ,00249,7		2202	1958	1713	1468	
3.13	1,80341 ,00232,7		0233	0465	0698	0931	1164	3.38	1,89804 ,00249,7		0245	0490	0736	0981	1226
	2094			1862	1629	1396			1,90042 ,00249,7		2207	1962	1716	1471	
3.14	1,80573 ,00233,2		0233	0466	0700	0933	1166	3.39	1,90349 ,00249,8		0246	0492	0737	0983	1229
	2099			1866	1632	1399			1,90587 ,00249,8		2212	1966	1721	1475	
3.15	1,80806 ,00233,6		0234	0467	0701	0934	1168	3.40	1,90825 ,00249,2		0246	0492	0739	0985	1231
	2102			1869	1635	1402			1,91163 ,00249,2		2216	1970	1723	1477	
3.16	1,81040 ,00234,2		0234	0468	0703	0937	1171	3.41	1,91501 ,00249,9		0247	0494	0741	0988	1235
	2108			1874	1639	1405			1,91739 ,00249,9		2222	1975	1728	1481	
3.17	1,81274 ,00234,5		0235	0469	0704	0938	1173	3.42	1,91977 ,00247,4		0247	0495	0742	0990	1237
	2111			1876	1642	1407			1,92215 ,00247,4		2227	1979	1732	1484	
3.18	1,81509 ,00235,0		0235	0470	0705	0940	1175	3.43	1,92553 ,00247,9		0248	0496	0744	0992	1240
	2115			1880	1645	1410			1,92791 ,00247,9		2231	1983	1735	1487	
3.19	1,81744 ,00235,5		0236	0471	0707	0942	1178	3.44	1,93029 ,00248,5		0249	0497	0746	0994	1243
	2120			1884	1649	1413			1,93267 ,00248,5		2237	1988	1740	1491	
3.20	1,81979 ,00236,0		0236	0472	0708	0944	1180	3.45	1,93605 ,00249,1		0249	0498	0747	0996	1246
	2124			1888	1652	1416			1,93843 ,00249,1		2242	1993	1744	1495	
3.21	1,82215 ,00236,5		0237	0473	0710	0946	1183	3.46	1,94081 ,00249,7		0250	0499	0749	0999	1248
	2129			1892	1656	1419			1,94319 ,00249,7		2247	1998	1748	1498	
3.22	1,82452 ,00237,0		0237	0474	0711	0948	1185	3.47	1,94557 ,00250,2		0250	0500	0751	1001	1251
	2133			1896	1659	1422			1,94795 ,00250,2		2252	2002	1751	1501	
3.23	1,82689 ,00237,4		0237	0475	0712	0950	1187	3.48	1,95033 ,00250,8		0251	0502	0752	1003	1254
	2137			1899	1662	1424			1,95271 ,00250,8		2257	2006	1756	1505	
3.24	1,82926 ,00238,0		0238	0476	0714	0952	1190	3.49	1,95509 ,00251,4		0251	0503	0754	1006	1257
	2142			1904	1666	1428			1,95747 ,00251,4		2263	2011	1760	1508	

General Table II.  $\lambda(a^7), \lambda\left(\frac{1-a^7}{a^7-1}\right)$ .

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$		1 9	2 8	3 7	4 6	5
3.50	1,89283 ,00252,0	,00	0252	0504	0756	1008	1260	3.75	1,95767 ,00268,0	,00	0268	0536	0804	1072	1340
3.51	1,89535 ,00252,6		0253	0505	0758	1010	1263	3.76	1,96035 ,00268,5		0269	0537	0806	1074	1343
3.52	1,89787 ,00253,1		0253	0506	0759	1012	1266	3.77	1,96303 ,00269,3		0269	0539	0808	1077	1347
3.53	1,90040 ,00253,7		0254	0507	0761	1015	1269	3.78	1,96573 ,00270,0		0270	0540	0810	1080	1350
3.54	1,90294 ,00254,3		0254	0509	0763	1017	1272	3.79	1,96843 ,00270,7		0271	0541	0812	1083	1354
3.55	1,90548 ,00255,0		0255	0510	0765	1020	1275	3.80	1,97113 ,00271,3		0271	0543	0814	1085	1357
3.56	1,90803 ,00255,5		0256	0511	0767	1022	1278	3.81	1,97385 ,00272,1		0272	0544	0816	1088	1361
3.57	1,91059 ,00256,2		0256	0512	0769	1025	1281	3.82	1,97657 ,00272,9		0273	0546	0819	1092	1365
3.58	1,91315 ,00256,8		0257	0514	0770	1027	1284	3.83	1,97930 ,00273,4		0273	0547	0820	1094	1367
3.59	1,91572 ,00257,4		0257	0515	0772	1030	1287	3.84	1,98203 ,00274,3		0274	0549	0823	1097	1372
3.60	1,91829 ,00258,0		0258	0516	0774	1032	1290	3.85	1,98477 ,00275,0		0275	0550	0825	1100	1375
3.61	1,92087 ,00258,7		0259	0517	0776	1035	1294	3.86	1,98752 ,00275,8		0276	0552	0827	1103	1379
3.62	1,92346 ,00259,2		0259	0518	0778	1037	1296	3.87	1,99028 ,00276,5		0277	0553	0830	1106	1383
3.63	1,92605 ,00259,9		0260	0520	0780	1040	1300	3.88	1,99305 ,00277,3		0277	0555	0832	1109	1387
3.64	1,92865 ,00260,6		0261	0521	0782	1042	1303	3.89	1,99582 ,00278,0		0278	0556	0834	1112	1390
3.65	1,93125 ,00261,2		0261	0522	0784	1045	1306	3.90	1,99860 ,00278,8		0279	0558	0836	1115	1394
3.66	1,93387 ,00261,8		0262	0524	0785	1047	1309	3.91	1,00139 ,00279,5		0280	0559	0839	1118	1398
3.67	1,93648 ,00262,5		0263	0525	0788	1050	1313	3.92	1,00418 ,00280,3		0280	0561	0841	1121	1402
3.68	1,93911 ,00263,2		0263	0526	0790	1053	1316	3.93	1,00699 ,00281,1		0281	0562	0843	1124	1406
3.69	1,94174 ,00263,8		0264	0528	0791	1055	1319	3.94	1,00980 ,00281,9		0282	0564	0846	1128	1410
3.70	1,94438 ,00264,4		0264	0529	0793	1058	1322	3.95	1,01262 ,00282,7		0283	0565	0848	1131	1414
3.71	1,94702 ,00265,1		0265	0530	0795	1060	1326	3.96	1,01544 ,00283,4		0283	0567	0850	1134	1417
3.72	1,94967 ,00265,8		0266	0532	0797	1063	1329	3.97	1,01828 ,00284,2		0284	0568	0853	1137	1421
3.73	1,95233 ,00266,5		0267	0533	0800	1066	1333	3.98	1,02112 ,00285,0		0285	0570	0855	1140	1425
3.74	1,95500 ,00267,2		0267	0534	0802	1069	1336	3.99	1,02397 ,00285,9		0286	0572	0858	1144	1430
			2405	2138	1870	1603					2573	2287	2001	1715	

General Table II.  $\lambda(a^t)$ ,  $\lambda\left(\frac{1-a^t}{a^{t-1}-1}\right)$ .

$\lambda(a^t)$	$\lambda\left(\frac{1-a^t}{a^{t-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^t)$	$\lambda\left(\frac{1-a^t}{a^{t-1}-1}\right)$		1 9	2 8	3 7	4 6	5
2.00	,02683 ,00286,7	,00	0287 2580	0573 2294	0860 2007	1147 1720	1434	2.25 2.26	,10107 ,00308,8	,00	0309 2779	0618 2470	0926 2162	1235 1853	1544
2.01	,02969 ,00287,5		0288 2588	0575 2300	0863 2013	1150 1725	1438	2.27 2.28	,10416 ,00309,8		0310 2788	0620 2478	0929 2169	1239 1859	1549
2.02	,03257 ,00288,3		0288 2595	0577 2306	0865 2018	1153 1730	1442	2.27 2.28	,10726 ,00310,7		0311 2796	0621 2486	0932 2175	1243 1864	1554
2.03	,03545 ,00289,1		0289 2602	0578 2313	0867 2024	1156 1735	1446	2.28 2.29	,11037 ,00311,7		0312 2805	0623 2494	0935 2182	1247 1870	1559
2.04	,03834 ,00290,0		0290 2610	0580 2320	0870 2030	1160 1740	1450	2.29 2.30	,11348 ,00312,7		0313 2814	0625 2502	0938 2189	1251 1876	1564
2.05	,04124 ,00290,8		0291 2617	0582 2326	0872 2036	1163 1745	1454	2.30 2.31	,11661 ,00313,7		0314 2823	0627 2510	0941 2196	1255 1882	1569
2.06	,04415 ,00291,7		0292 2625	0583 2334	0875 2043	1167 1750	1459	2.31 2.32	,11975 ,00314,6		0315 2831	0629 2517	0944 2202	1258 1888	1573
2.07	,04707 ,00292,5		0293 2633	0585 2340	0878 2048	1170 1755	1463	2.32 2.33	,12289 ,00315,6		0316 2840	0631 2525	0947 2209	1262 1894	1578
2.08	,04999 ,00293,4		0293 2641	0587 2347	0880 2054	1174 1760	1467	2.33 2.34	,12605 ,00316,7		0317 2850	0633 2534	0950 2216	1267 1900	1583
2.09	,05293 ,00294,2		0294 2648	0588 2354	0883 2059	1177 1765	1471	2.34 2.35	,12921 ,00317,7		0318 2859	0635 2542	0953 2224	1271 1906	1589
2.10	,05587 ,00295,1		0295 2656	0590 2361	0885 2066	1180 1771	1476	2.35 2.36	,13239 ,00318,7		0319 2868	0637 2550	0956 2231	1275 1912	1594
2.11	,05882 ,00296,0		0296 2664	0592 2368	0888 2072	1184 1776	1480	2.36 2.37	,13558 ,00319,7		0320 2877	0639 2558	0959 2238	1279 1918	1599
2.12	,06178 ,00296,8		0297 2671	0594 2374	0890 2078	1187 1781	1484	2.37 2.38	,13898 ,00320,7		0321 2886	0641 2566	0962 2245	1283 1924	1604
2.13	,06475 ,00297,8		0298 2680	0596 2382	0893 2085	1191 1787	1489	2.38 2.39	,14198 ,00321,8		0322 2896	0644 2574	0965 2253	1287 1931	1609
2.14	,06772 ,00298,6		0299 2687	0597 2389	0896 2090	1194 1792	1493	2.39 2.40	,14520 ,00322,8		0323 2905	0646 2582	0968 2260	1291 1937	1614
2.15	,07071 ,00299,5		0300 2696	0599 2396	0899 2097	1198 1797	1498	2.40 2.41	,14843 ,00323,9		0324 2915	0648 2591	0972 2267	1296 1943	1620
2.16	,07370 ,00300,4		0300 2704	0601 2403	0901 2103	1202 1802	1502	2.41 2.42	,15167 ,00324,9		0325 2924	0650 2599	0975 2274	1300 1949	1625
2.17	,07671 ,00301,3		0301 2712	0603 2410	0904 2109	1205 1808	1507	2.42 2.43	,15492 ,00325,9		0326 2933	0652 2607	0978 2281	1304 1955	1630
2.18	,07972 ,00302,2		0302 2720	0604 2418	0907 2115	1209 1813	1511	2.43 2.44	,15818 ,00327,1		0327 2944	0654 2617	0981 2290	1308 1963	1636
2.19	,08274 ,00303,1		0303 2728	0606 2425	0909 2122	1212 1819	1516	2.44 2.45	,16145 ,00328,1		0328 2953	0656 2625	0984 2297	1312 1969	1641
2.20	,08577 ,00304,1		0304 2737	0608 2433	0912 2129	1216 1825	1521	2.45 2.46	,16473 ,00329,2		0329 2963	0658 2634	0988 2304	1317 1975	1646
2.21	,08882 ,00305,0		0305 2745	0610 2440	0915 2135	1220 1830	1525	2.46 2.47	,16802 ,00330,3		0330 2973	0661 2642	0991 2312	1321 1982	1652
2.22	,09187 ,00305,9		0306 2753	0612 2447	0918 2141	1224 1835	1530	2.47 2.48	,17132 ,00331,4		0331 2983	0663 2651	0994 2320	1326 1988	1657
2.23	,09493 ,00306,9		0307 2762	0614 2455	0921 2148	1228 1841	1535	2.48 2.49	,17464 ,00332,5		0333 2993	0665 2660	0998 2328	1330 1995	1663
2.24	,09799 ,00307,8		0308 2770	0616 2462	0923 2155	1231 1847	1539	2.49 2.50	,17796 ,00333,6		0334 3002	0667 2669	1001 2335	1334 2002	1668

General Table II.  $\lambda(a^r)$ ,  $\lambda\left(\frac{1-a^r}{a^r-1}\right)$ .

$\lambda(a^r)$	$\lambda\left(\frac{1-a^r}{a^r-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^r)$	$\lambda\left(\frac{1-a^r}{a^r-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{z}.50$	,18130 ,00334,7	,00	0335	0669	1004	1339	1674	$\bar{z}.75$	,26850 ,00364,7	,00	0365	0729	1094	1459	1824
$\bar{z}.51$	,18464 ,00335,8	0336	0672	1007	1343	1679	$\bar{z}.76$	,27214 ,00366,2	0366	0732	1099	1465	1831		
$\bar{z}.52$	,18880 ,00336,9	0337	0674	1011	1348	1685	$\bar{z}.77$	,27581 ,00367,4	0367	0735	1102	1470	1837		
$\bar{z}.53$	,19137 ,00338,1	0338	0676	1014	1352	1691	$\bar{z}.78$	,27948 ,00368,7	0369	0737	1106	1475	1844		
$\bar{z}.54$	,19475 ,00339,2	0339	0678	1018	1357	1696	$\bar{z}.79$	,28317 ,00370,1	0370	0740	1110	1480	1851		
$\bar{z}.55$	,19815 ,00340,4	0340	0681	1021	1362	1702	$\bar{z}.80$	,28687 ,00371,3	0370	0743	1114	1485	1857		
$\bar{z}.56$	,20155 ,00341,5	0342	0683	1025	1366	1708	$\bar{z}.81$	,29058 ,00372,8	0373	0746	1118	1491	1864		
$\bar{z}.57$	,20496 ,00342,7	0343	0685	1028	1371	1714	$\bar{z}.82$	,29431 ,00373,9	0374	0748	1122	1496	1870		
$\bar{z}.58$	,20839 ,00343,9	0344	0688	1032	1376	1720	$\bar{z}.83$	,29805 ,00375,4	0375	0751	1126	1502	1877		
$\bar{z}.59$	,21183 ,00345,0	0345	0690	1035	1380	1725	$\bar{z}.84$	,30180 ,00376,7	0377	0753	1130	1507	1884		
$\bar{z}.60$	,21528 ,00346,2	0346	0692	1039	1385	1731	$\bar{z}.85$	,30557 ,00378,1	0378	0756	1134	1512	1891		
$\bar{z}.61$	,21874 ,00347,4	0347	0695	1042	1390	1737	$\bar{z}.86$	,30935 ,00379,4	0379	0759	1138	1518	1897		
$\bar{z}.62$	,22222 ,00348,7	0349	0697	1046	1395	1744	$\bar{z}.87$	,31314 ,00380,8	0381	0762	1142	1523	1904		
$\bar{z}.63$	,22570 ,00349,7	0350	0699	1049	1399	1749	$\bar{z}.88$	,31695 ,00382,2	0382	0764	1146	1529	1911		
$\bar{z}.64$	,22920 ,00351,1	0351	0702	1053	1404	1756	$\bar{z}.89$	,32077 ,00383,6	0384	0767	1151	1534	1918		
$\bar{z}.65$	,23271 ,00352,3	0352	0705	1057	1409	1762	$\bar{z}.90$	,32461 ,00385,0	0385	0770	1155	1540	1925		
$\bar{z}.66$	,23623 ,00353,5	0354	0707	1061	1414	1768	$\bar{z}.91$	,32846 ,00386,3	0386	0773	1159	1545	1932		
$\bar{z}.67$	,23977 ,00354,7	0355	0709	1064	1419	1774	$\bar{z}.92$	,33232 ,00387,8	0388	0776	1163	1551	1939		
$\bar{z}.68$	,24332 ,00356,0	0356	0712	1068	1424	1780	$\bar{z}.93$	,33620 ,00389,2	0389	0778	1168	1557	1946		
$\bar{z}.69$	,24688 ,00357,2	0357	0714	1072	1429	1786	$\bar{z}.94$	,34009 ,00390,6	0391	0781	1172	1562	1953		
$\bar{z}.70$	,25045 ,00358,4	0358	0717	1075	1434	1792	$\bar{z}.95$	,34400 ,00392,0	0392	0784	1176	1568	1960		
$\bar{z}.71$	,25403 ,00359,7	0360	0719	1079	1439	1799	$\bar{z}.96$	,34792 ,00393,5	0394	0787	1181	1574	1968		
$\bar{z}.72$	,25763 ,00361,0	0361	0722	1083	1444	1805	$\bar{z}.97$	,35185 ,00394,9	0395	0790	1185	1580	1975		
$\bar{z}.73$	,26124 ,00362,2	0362	0724	1087	1449	1811	$\bar{z}.98$	,35580 ,00396,3	0396	0793	1189	1585	1982		
$\bar{z}.74$	,26486 ,00363,5	0364	0727	1091	1454	1817	$\bar{z}.99$	,35976 ,00397,8	0398	0796	1193	1591	1989		
		3272	2908	2545	2181				3580	3182	2785	2387			

General Table II.  $\lambda(a^t)$ ,  $\lambda\left(\frac{1-a^t}{a^{-1}-1}\right)$ .

$\lambda(a^t)$	$\lambda\left(\frac{1-a^t}{a^{-1}-1}\right)$		1	2	3	4	5	$\lambda(a^t)$	$\lambda\left(\frac{1-a^t}{a^{-1}-1}\right)$		1	2	3	4	5
1.00	36374	,00	0399	0799	1198	1597	1997	1.25	,48811	,00	0438	0876	1314	1752	2190
,00399,3	3594	3194	2795	2396					,00438,0	3942	3504	3066	2628		
1.01	36773	0401	0801	1202	1603	2004		1.26	,47249	0439	0879	1318	1758	2197	
,00400,7	3606	3206	2805	2404					,00439,4	3955	3515	3076	2636		
1.02	37174	0402	0804	1207	1609	2011		1.27	,47689	0441	0882	1324	1765	2206	
,00402,2	3620	3218	2815	2413					,00441,2	3971	3530	3088	2647		
1.03	37576	0404	0807	1211	1615	2019		1.28	,48130	0443	0886	1329	1772	2215	
,00403,7	3633	3230	2826	2422					,00442,9	3986	3543	3100	2657		
1.04	37980	0405	0811	1216	1622	2027		1.29	,48573	0445	0889	1334	1778	2223	
,00405,4	3649	3243	2838	2432					,00444,5	4001	3556	3112	2667		
1.05	38385	0406	0813	1219	1626	2032		1.30	,49017	0446	0892	1339	1785	2231	
,00406,4	3658	3251	2845	2438					,00446,2	4016	3579	3123	2677		
1.06	38792	0408	0816	1225	1633	2041		1.31	,49463	0448	0896	1343	1791	2239	
,00408,2	3674	3266	2857	2449					,00447,8	4030	3582	3135	2687		
1.07	39200	0410	0819	1229	1639	2050		1.32	,49911	0450	0899	1349	1798	2248	
,00409,7	3687	3278	2868	2458					,00449,5	4046	3596	3147	2697		
1.08	39610	0411	0822	1234	1645	2056		1.33	,50361	0451	0902	1353	1804	2256	
,00411,2	3701	3290	2878	2467					,00451,1	4060	3609	3158	2707		
1.09	40021	0413	0826	1238	1651	2064		1.34	,50812	0453	0909	1359	1812	2265	
,00412,8	3715	3302	2890	2477					,00452,9	4076	3623	3170	2717		
1.10	40434	0414	0828	1243	1657	2071		1.35	,51265	0455	0909	1364	1818	2273	
,00414,2	3728	3314	2899	2485					,00454,5	4090	3636	3182	2727		
1.11	40848	0416	0832	1247	1663	2079		1.36	,51719	0456	0912	1369	1825	2281	
,00415,8	3742	3326	2911	2495					,00456,2	4106	3650	3193	2737		
1.12	41264	0417	0835	1252	1669	2087		1.37	,52175	0458	0916	1373	1831	2289	
,00417,3	3756	3338	2921	2504					,00457,8	4120	3662	3205	2747		
1.13	41681	0419	0838	1257	1676	2096		1.38	,52633	0460	0919	1379	1839	2299	
,00419,1	3772	3353	2934	2515					,00459,7	4137	3678	3218	2758		
1.14	42100	0420	0840	1261	1681	2101		1.39	,53093	0461	0923	1384	1846	2307	
,00420,2	3782	3362	2941	2521					,00461,4	4153	3691	3230	2768		
1.15	42520	0422	0844	1266	1688	2110		1.40	,53554	0463	0926	1389	1852	2315	
,00421,9	3797	3375	2953	2531					,00462,9	4166	3703	3240	2777		
1.16	42942	0424	0847	1271	1694	2118		1.41	,54017	0465	0930	1394	1859	2324	
,00423,6	3812	3389	2965	2542					,00464,8	4183	3718	3254	2789		
1.17	43366	0425	0850	1275	1700	2126		1.42	,54482	0467	0933	1400	1866	2333	
,00425,1	3826	3401	2976	2551					,00466,5	4199	3732	3266	2799		
1.18	43791	0427	0854	1280	1707	2134		1.43	,54948	0468	0936	1404	1872	2341	
,00426,8	3841	3414	2988	2561					,00468,1	4213	3745	3277	2809		
1.19	44218	0428	0856	1284	1712	2141		1.44	,55417	0470	0940	1410	1880	2350	
,00428,1	3853	3425	2997	2569					,00469,9	4229	3759	3289	2819		
1.20	44646	0430	0860	1290	1720	2150		1.45	,55886	0472	0943	1415	1887	2359	
,00429,9	3869	3439	3009	2579					,00471,7	4245	3774	3302	2830		
1.21	45076	0432	0863	1295	1726	2158		1.46	,56358	0474	0947	1421	1894	2368	
,00431,5	3884	3452	3021	2589					,00473,6	4262	3789	3315	2842		
1.22	45507	0433	0866	1299	1732	2166		1.47	,56832	0475	0950	1425	1900	2376	
,00433,1	3898	3465	3032	2599					,00475,1	4276	3801	3326	2851		
1.23	45940	0435	0869	1304	1739	2174		1.48	,57307	0477	0954	1431	1901	2385	
,00434,7	3912	3478	3043	2608					,00476,9	4292	3815	3338	2861		
1.24	46375	0436	0873	1309	1745	2182		1.49	,57784	0479	0957	1436	1915	2394	
,00436,3	3927	3490	3054	2618					,00478,7	4308	3830	3351	2872		

General Table II.  $\lambda(a^7), \lambda\left(\frac{1-a^7}{a^7-1}\right)$ .

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$	1	2	3	4	5	$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$	1	2	3	4	5	
		9	8	7	6	5			9	8	7	6	5	
1.50	,58262 ,00480,2	,00	0480	0960	1441	1921	2401	1.75 ,00	,70810 ,00526,0	0526	1052	1578	2104	2630
1.51	,58742 ,00482,2		4322	3842	3361	2881	2411	1.76 ,00	,71336 ,00527,3	4734	4208	3682	3156	
1.52	,59225 ,00483,9		0482	0964	1447	1929	2411	1.77 ,00	,71863 ,00529,6	0527	1055	1582	2109	2637
1.53	,59709 ,00485,7		4340	3858	3375	2893		1.78 ,00	,72392 ,00531,0	4746	4218	3691	3164	
1.54	,60194 ,00487,4		0486	0971	1457	1943	2429	1.79 ,00	,72924 ,00533,3	0530	1059	1589	2118	2648
			4371	3886	3400	2914	2437			4766	4237	3707	3178	
			0487	0975	1462	1950	2437			0531	1062	1593	2124	2655
			4387	3899	3412	2924				4779	4248	3717	3186	
										0533	1067	1600	2133	2667
										4800	4266	3733	3200	
1.55	,60682 ,00489,3		0489	0979	1468	1957	2447	1.80 ,00	,73457 ,00534,7	0535	1069	1604	2139	2674
1.56	,61171 ,00491,1		4404	3914	3425	2936		1.81 ,00	,73992 ,00536,1	4812	4278	3743	3208	
1.57	,61662 ,00492,7		0491	0982	1473	1964	2456	1.82 ,00	,74528 ,00538,5	0536	1072	1608	2144	2681
1.58	,62155 ,00494,6		4420	3929	3438	2947		1.83 ,00	,75067 ,00540,7	4825	4289	3753	3217	
1.59	,62649 ,00496,4		0493	0985	1478	1971	2464	1.84 ,00	,75608 ,00542,3	0539	1077	1616	2154	2693
			4434	3942	3449	2956				4847	4308	3770	3231	
			0496	0993	1489	1986	2482			0541	1081	1622	2163	2704
			4468	3971	3475	2978				4866	4326	3785	3244	
										0542	1085	1627	2169	2712
										4881	4338	3796	3254	
1.60	,63146 ,00498,3		0498	0997	1495	1993	2492	1.85 ,00	,76150 ,00544,1	0544	1088	1632	2176	2721
1.61	,63644 ,00500,0		4485	3986	3488	2990		1.86 ,00	,76694 ,00546,1	4897	4353	3809	3265	
1.62	,64144 ,00501,8		0500	1000	1500	2000	2500	1.87 ,00	,77240 ,00548,1	0546	1092	1638	2184	2731
1.63	,64646 ,00503,7		4500	4000	3500	3000		1.88 ,00	,77788 ,00550,0	4915	4369	3823	3277	
1.64	,65150 ,00505,3		0502	1004	1505	2007	2509	1.89 ,00	,78338 ,00551,4	0548	1096	1644	2192	2741
			4516	4014	3513	3011				4933	4385	3837	3289	
			0504	1007	1511	2015	2519			0550	1100	1650	2200	2750
			4533	4030	3526	3022				4950	4400	3850	3300	
			0505	1011	1516	2021	2527			0551	1103	1654	2206	2757
			4548	4042	3537	3032				4963	4411	3860	3308	
1.65	,65655 ,00507,4		0507	1015	1522	2030	2537	1.90 ,00	,78889 ,00553,8	0554	1108	1661	2215	2769
1.66	,66162 ,00509,1		4567	4059	3552	3044		1.91 ,00	,79443 ,00555,5	4984	4430	3877	3323	
1.67	,66671 ,00511,0		0509	1018	1527	2036	2546	1.92 ,00	,79999 ,00557,3	0556	1111	1667	2222	2778
1.68	,67182 ,00512,7		4582	4073	3564	3055		1.93 ,00	,80556 ,00559,5	5000	4444	3889	3333	
1.69	,67695 ,00514,6		0511	1022	1533	2044	2555	1.94 ,00	,81116 ,00561,0	0557	1115	1672	2229	2787
			4599	4088	3577	3066				5016	4458	3901	3344	
			0513	1025	1538	2051	2564			0560	1119	1679	2238	2798
			4614	4102	3589	3076				5036	4476	3917	3357	
			0515	1029	1544	2058	2573			0561	1122	1683	2244	2805
			4631	4117	3602	3088				5049	4488	3927	3366	
1.70	,68209 ,00516,5		0517	1033	1550	2066	2583	1.95 ,00	,81677 ,00563,2	0563	1126	1690	2253	2816
1.71	,68726 ,00518,1		4649	4132	3616	3099		1.96 ,00	,82240 ,00564,6	5069	4506	3942	3379	
1.72	,69244 ,00520,2		0518	1036	1554	2072	2591	1.97 ,00	,82804 ,00568,4	0565	1129	1694	2258	2823
1.73	,69764 ,00522,0		4663	4145	3627	3089		1.98 ,00	,83373 ,00568,3	5081	4517	3952	3388	
1.74	,70286 ,00523,7		0524	1047	1571	2095	2619	1.99 ,00	,83941	0568	1137	1705	2273	2842
			4713	4190	3666	3142				5115	4546	3978	3410	

General Table III.  $\lambda(a^s)$ ,  $\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$ .

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
3.00	1,52519 ,00266,3	,00	0266	0533	0799	1065	1332	3.25	1,59300 ,00276,9	,00	0277	0554	0831	1108	1385
3.01	1,52786 ,00266,5		0267	0533	0800	1066	1333	3.26	1,59577 ,00277,4		0277	0555	0832	1110	1387
3.02	1,53052 ,00267,2		0267	0534	0802	1069	1336	3.27	1,59855 ,00277,9		0278	0556	0834	1112	1390
3.03	1,53319 ,00267,5		0268	0535	0803	1070	1338	3.28	1,60133 ,00278,4		0278	0557	0835	1114	1392
3.04	1,53587 ,00267,9		0268	0536	0804	1072	1340	3.29	1,60411 ,00278,8		0279	0558	0836	1115	1394
3.05	1,53855 ,00268,3		0268	0537	0805	1073	1342	3.30	1,60690 ,00279,3		0279	0559	0838	1117	1397
3.06	1,54123 ,00268,7		0269	0537	0806	1075	1344	3.31	1,60969 ,00279,8		0280	0560	0839	1119	1399
3.07	1,54392 ,00269,1		0269	0538	0807	1076	1346	3.32	1,61249 ,00280,2		0280	0560	0841	1121	1401
3.08	1,54661 ,00269,5		0270	0539	0809	1078	1348	3.33	1,61529 ,00280,8		0281	0562	0842	1123	1404
3.09	1,54930 ,00269,9		0270	0540	0810	1080	1350	3.34	1,61810 ,00281,3		0281	0563	0844	1125	1407
3.10	1,55200 ,00270,3		0270	0541	0811	1081	1352	3.35	1,62091 ,00281,8		0282	0564	0845	1127	1409
3.11	1,55470 ,00270,7		0271	0541	0812	1083	1354	3.36	1,62373 ,00282,2		0282	0564	0847	1129	1411
3.12	1,55741 ,00271,2		0271	0542	0814	1085	1356	3.37	1,62655 ,00282,8		0283	0566	0848	1131	1414
3.13	1,56012 ,00271,6		0272	0543	0815	1086	1358	3.38	1,62938 ,00283,3		0283	0567	0850	1133	1417
3.14	1,56284 ,00272,0		0272	0544	0816	1088	1360	3.39	1,63221 ,00283,8		0284	0568	0851	1135	1419
3.15	1,56556 ,00272,5		0273	0545	0818	1090	1363	3.40	1,63505 ,00284,3		0284	0569	0853	1137	1422
3.16	1,56828 ,00272,9		0273	0546	0819	1092	1365	3.41	1,63789 ,00284,8		0285	0570	0854	1139	1424
3.17	1,57101 ,00273,3		0273	0547	0820	1093	1367	3.42	1,64074 ,00285,3		0285	0571	0856	1141	1427
3.18	1,57375 ,00273,8		0274	0548	0821	1095	1369	3.43	1,64359 ,00285,8		0286	0572	0857	1143	1429
3.19	1,57648 ,00274,2		0274	0548	0823	1097	1371	3.44	1,64645 ,00286,3		0286	0573	0859	1145	1432
3.20	1,57923 ,00274,7		0275	0549	0824	1099	1374	3.45	1,64931 ,00286,9		0287	0574	0861	1148	1435
3.21	1,58197 ,00275,1		0275	0550	0825	1100	1376	3.46	1,65218 ,00287,4		0287	0575	0862	1150	1437
3.22	1,58472 ,00275,6		0276	0551	0827	1102	1378	3.47	1,65506 ,00288,0		0288	0576	0864	1152	1440
3.23	1,58748 ,00276,0		0276	0552	0828	1104	1380	3.48	1,65794 ,00288,5		0289	0577	0866	1154	1443
3.24	1,59024 ,00276,5		0277	0553	0830	1106	1383	3.49	1,66082 ,00289,0		0289	0578	0867	1156	1445
			2489	2212	1936	1659					2601	2312	2023	1734	

General Table III.  $\lambda(a^s), \lambda\left(\frac{1-a^s}{a^s-1}\right)$ .

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^s-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^s-1}\right)$		1 9	2 8	3 7	4 6	5
3.50	1,66371 ,00289,6	,00	0290	0579	0869	1158	1448	3.75	1,73787 ,00304,7	,00	0305	0609	0914	1219	1524
3.51	1,66661 ,00290,1	0290	0580	0870	1160	1451	3.76	1,74091 ,00305,3	0305	0611	0916	1221	1527		
3.52	1,66951 ,00290,7	0291	0581	0872	1163	1454	3.77	1,74393 ,00306,0	0306	0612	0918	1224	1530		
3.53	1,67274 ,00291,4	0291	0583	0874	1166	1457	3.78	1,74702 ,00306,6	0307	0613	0920	1226	1533		
3.54	1,67533 ,00291,8	0292	0584	0875	1167	1459	3.79	1,75009 ,00307,4	0307	0615	0922	1230	1537		
3.55	1,67825 ,00292,4	0292	0585	0877	1170	1462	3.80	1,75316 ,00308,0	0308	0616	0924	1232	1540		
3.56	1,68117 ,00292,9	0293	0586	0879	1172	1465	3.81	1,75624 ,00308,8	0309	0618	0926	1235	1544		
3.57	1,68410 ,00293,7	0294	0587	0881	1175	1469	3.82	1,75933 ,00309,3	0309	0609	0928	1237	1547		
3.58	1,68704 ,00294,1	0294	0588	0882	1176	1471	3.83	1,76243 ,00310,1	0310	0620	0930	1240	1551		
3.59	1,68998 ,00294,7	0295	0589	0884	1179	1474	3.84	1,76553 ,00310,8	0311	0622	0932	1243	1554		
3.60	1,69293 ,00295,3	0295	0591	0886	1181	1477	3.85	1,76863 ,00311,4	0311	0623	0934	1246	1557		
3.61	1,69588 ,00295,9	0296	0592	0888	1184	1480	3.86	1,77175 ,00312,2	0312	0624	0937	1249	1561		
3.62	1,69884 ,00296,5	0297	0593	0890	1186	1483	3.87	1,77487 ,00313,0	0313	0626	0939	1252	1565		
3.63	1,70180 ,00297,1	0297	0594	0891	1188	1486	3.88	1,77800 ,00313,6	0314	0627	0941	1254	1568		
3.64	1,70477 ,00297,7	0298	0595	0893	1191	1489	3.89	1,78113 ,00314,3	0314	0629	0943	1257	1572		
3.65	1,70775 ,00298,4	0298	0597	0895	1194	1492	3.90	1,78428 ,00315,0	0315	0630	0945	1260	1575		
3.66	1,71074 ,00298,88	0299	0598	0896	1195	1494	3.91	1,78743 ,00315,8	0316	0632	0947	1263	1579		
3.67	1,71372 ,00299,6	0300	0599	0899	1198	1498	3.92	1,79059 ,00316,4	0316	0633	0949	1266	1582		
3.68	1,71672 ,00300,2	0300	0600	0901	1201	1501	3.93	1,79375 ,00317,3	0317	0635	0952	1269	1587		
3.69	1,71972 ,00300,8	0301	0602	0902	1203	1504	3.94	1,79692 ,00318,0	0318	0636	0954	1272	1590		
3.70	1,72273 ,00301,4	0301	0603	0904	1206	1507	3.95	1,80010 ,00318,7	0319	0637	0956	1275	1594		
3.71	1,72574 ,00302,0	0302	0604	0906	1208	1510	3.96	1,80329 ,00319,5	0320	0639	0959	1278	1598		
3.72	1,72876 ,00302,7	0303	0605	0908	1211	1514	3.97	1,80648 ,00320,3	0320	0641	0961	1281	1602		
3.73	1,73179 ,00303,4	0303	0607	0910	1214	1517	3.98	1,80969 ,00321,0	0321	0642	0963	1284	1605		
3.74	1,73483 ,00304,0	0304	0608	0912	1216	1520	3.99	1,81290 ,00321,8	0322	0644	0965	1287	1609		
		2730	2432	2128	1824				2896	2574	2253	1931			

General Table III.  $\lambda(a^s), \lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$ .

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
2.00	1,81612 ,00322,5	,00	0323	0645	0968	1290	1613	2.25	1,89923 ,00343,8	,00	0344	0688	1031	1375	1719
2.01	1,81934 ,00323,4		2903	2580	2258	1935	1617	2.26	1,90207 ,00344,7		3094	2750	2407	2063	
2.02	1,82257 ,00324,2		0323	0647	0970	1294	1617	2.27	1,90612 ,00345,7		0345	0689	1034	1379	1724
2.03	1,82582 ,00324,9		2911	2587	2264	1940		2.28	1,90958 ,00346,6		0346	0691	1037	1383	1729
2.04	1,82907 ,00325,8		0324	0648	0973	1297	1621	2.29	1,91304 ,00347,6		0347	0693	1040	1386	1733
2.05	1,83232 ,00326,5		0325	0650	0975	1300	1625	2.30	1,91652 ,00348,5		0349	0697	1046	1394	1743
2.06	1,83559 ,00327,3		2918	2594	2269	1945		2.31	1,92000 ,00349,3		3137	2788	2440	2091	
2.07	1,83886 ,00328,2		0326	0652	0977	1303	1629	2.32	1,92349 ,00350,7		0349	0699	1048	1397	1747
2.08	1,84214 ,00329,0		2924	2599	2274	1949		2.33	1,92700 ,00351,4		3144	2794	2445	2096	
2.09	1,84543 ,00329,8		2932	2606	2281	1955		2.34	1,93052 ,00352,4		0351	0701	1052	1403	1754
2.10	1,84873 ,00330,6		0327	0653	0980	1306	1633	2.35	1,93404 ,00353,3		3156	2806	2455	2104	
2.11	1,85204 ,00331,5		2939	2612	2286	1959		2.36	1,93757 ,00354,3		0351	0705	1057	1410	1762
2.12	1,85553 ,00332,4		0327	0655	0982	1309	1637	2.37	1,94112 ,00355,4		3172	2819	2467	2114	
2.13	1,85868 ,00333,2		2946	2618	2291	1964		2.38	1,94467 ,00356,4		0353	0707	1060	1413	1767
2.14	1,86201 ,00334,0		2954	2626	2297	1969		2.39	1,94823 ,00357,3		3180	2826	2473	2120	
2.15	1,86535 ,00334,9		0331	0661	0992	1322	1653	2.40	1,95181 ,00358,4		0354	0709	1063	1417	1771
2.16	1,86870 ,00335,7		2975	2645	2314	1984		2.41	1,95539 ,00359,4		3189	2834	2480	2126	
2.17	1,87205 ,00336,6		0332	0663	0995	1326	1658	2.42	1,95898 ,00360,4		0355	0711	1066	1422	1777
2.18	1,87542 ,00337,5		2992	2659	2327	1994		2.43	1,96259 ,00361,4		3199	2843	2488	2132	
2.19	1,87880 ,00338,4		0333	0666	1000	1333	1666	2.44	1,96620 ,00362,4		0356	0713	1069	1426	1782
2.20	1,88218 ,00339,3		3014	2679	2344	2009	1675	2.45	1,96983 ,00363,4		0358	0717	1075	1434	1792
2.21	1,88557 ,00340,2		0336	0671	1007	1343	1679	2.46	1,97346 ,00364,6		3226	2867	2509	2150	
2.22	1,88897 ,00341,0		3021	2686	2350	2014		2.47	1,97711 ,00365,6		0359	0719	1078	1438	1797
2.23	1,89238 ,00342,1		0337	0673	1010	1346	1683	2.48	1,98076 ,00366,7		0360	0721	1081	1442	1802
2.24	1,89581 ,00342,8		3046	2707	2369	2030		2.49	1,98443 ,00367,8		3244	2883	2523	2162	
2.25											0361	0723	1084	1446	1807
2.26											3253	2891	2530	2168	
2.27											0362	0725	1087	1450	1812
2.28											3262	2899	2537	2174	
2.29											0363	0727	1090	1454	1817
2.30											3271	2907	2544	2180	
2.31											0365	0729	1094	1458	1823
2.32											3281	2917	2552	2188	
2.33											0366	0731	1097	1462	1828
2.34											3290	2925	2559	2194	
2.35											0367	0733	1100	1467	1834
2.36											3300	2934	2567	2200	
2.37											0368	0735	1103	1471	1839
2.38											3310	2942	2575	2207	

General Table III.  $\lambda(a^s)$ ,  $\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$ .

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1	2	3	4	5	$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1	2	3	4	5
			9	8	7	6	5				9	8	7	6	5
$\bar{2.50}$	$1,98811$	,00	0369	0738	1106	1475	1844	$\bar{2.75}$	,08373	,00	0398	0796	1194	1592	1990
	,00368,8		3319	2950	2582	2213			,00397,9		3581	3183	2785	2387	
$\bar{2.51}$	$1,99179$		0370	0740	1109	1480	1850	$\bar{2.76}$	,08771		0399	0799	1198	1598	1997
	,00369,9		3329	2959	2589	2219			,00399,4		3595	3195	2796	2396	
$\bar{2.52}$	$1,99549$		0371	0742	1113	1484	1855	$\bar{2.77}$	,09170		0401	0801	1202	1602	2003
	,00371,0		3339	2968	2597	2226			,00400,6		3605	3205	2804	2404	
$\bar{2.53}$	$1,99920$		0372	0744	1116	1488	1861	$\bar{2.78}$	,09571		0402	0803	1205	1607	2009
	,00372,1		3349	2977	2605	2233			,00401,7		3615	3214	2812	2410	
$\bar{2.54}$	$,00293$		0373	0747	1120	1493	1867	$\bar{2.79}$	,09972		0403	0806	1209	1612	2016
	,00373,3		3360	2986	2613	2240			,00403,1		3628	3225	2822	2419	
$\bar{2.55}$	,00666		0374	0749	1123	1497	1872	$\bar{2.80}$	,10376		0404	0809	1213	1618	2022
	,00374,3		3369	2994	2620	2246			,00404,4		3640	3235	2831	2426	
$\bar{2.56}$	,01040		0376	0751	1127	1502	1878	$\bar{2.81}$	,10780		0406	0811	1217	1623	2029
	,00375,5		3380	3004	2629	2253			,00405,7		3651	3246	2840	2434	
$\bar{2.57}$	,01416		0376	0753	1129	1506	1882	$\bar{2.82}$	,11186		0407	0814	1221	1628	2035
	,00376,4		3388	3011	2635	2258			,00407,0		3663	3256	2849	2442	
$\bar{2.58}$	,00792		0378	0756	1134	1512	1890	$\bar{2.83}$	,11593		0408	0817	1225	1633	2042
	,00377,9		3401	3023	2645	2267			,00408,3		3675	3266	2858	2450	
$\bar{2.59}$	,02170		0379	0758	1136	1515	1894	$\bar{2.84}$	,12001		0410	0819	1229	1638	2048
	,00378,3		3409	3030	2652	2273			,00409,5		3686	3276	2867	2457	
$\bar{2.60}$	,02549		0380	0760	1140	1520	1900	$\bar{2.85}$	,12410		0411	0822	1233	1644	2055
	,00380,0		3420	3040	2660	2280			,00410,9		3698	3287	2876	2465	
$\bar{2.61}$	,02929		0381	0762	1144	1525	1906	$\bar{2.86}$	,12821		0412	0824	1237	1649	2061
	,00381,2		3431	3050	2668	2287			,00412,2		3710	3298	2885	2473	
$\bar{2.62}$	,03310		0382	0765	1147	1530	1912	$\bar{2.87}$	,13234		0414	0827	1241	1654	2068
	,00382,4		3442	3059	2677	2294			,00413,6		3722	3309	2895	2482	
$\bar{2.63}$	,03692		0383	0767	1150	1534	1917	$\bar{2.88}$	,13647		0415	0830	1245	1660	2075
	,00383,4		3451	3067	2684	2300			,00414,9		3734	3319	2904	2489	
$\bar{2.64}$	,04076		0385	0769	1154	1539	1924	$\bar{2.89}$	,14062		0416	0833	1249	1665	2082
	,00384,7		3462	3078	2693	2308			,00416,3		3747	3330	2914	2498	
$\bar{2.65}$	,04460		0386	0772	1157	1543	1929	$\bar{2.90}$	,14478		0418	0835	1253	1670	2088
	,00385,8		3472	3086	2701	2315			,00417,6		3758	3341	2923	2506	
$\bar{2.66}$	,04846		0387	0774	1161	1548	1936	$\bar{2.91}$	,14896		0419	0838	1257	1676	2095
	,00387,1		3484	3097	2710	2323			,00419,0		3771	3352	2933	2514	
$\bar{2.67}$	,05233		0388	0776	1165	1553	1941	$\bar{2.92}$	,15315		0420	0841	1261	1682	2102
	,00388,2		3494	3106	2717	2329			,00420,4		3784	3363	2943	2522	
$\bar{2.68}$	,05621		0389	0779	1168	1558	1947	$\bar{2.93}$	,15735		0422	0843	1265	1687	2109
	,00389,4		3505	3115	2726	2336			,00421,7		3795	3344	2952	2530	
$\bar{2.69}$	,06010		0391	0781	1172	1562	1953	$\bar{2.94}$	,16157		0423	0846	1269	1692	2116
	,00390,6		3515	3125	2734	2344			,00423,1		3808	3385	2962	2539	
$\bar{2.70}$	,06401		0392	0784	1176	1568	1960	$\bar{2.95}$	,16580		0425	0849	1274	1698	2123
	,00391,9		3527	3135	2743	2351			,00424,6		3821	3397	2972	2548	
$\bar{2.71}$	,06793		0393	0786	1179	1572	1965	$\bar{2.96}$	,17005		0426	0852	1277	1703	2129
	,00393,0		3537	3144	2751	2358			,00425,8		3832	3406	2981	2555	
$\bar{2.72}$	,07186		0394	0789	1183	1577	1972	$\bar{2.97}$	,17430		0427	0855	1282	1719	2137
	,00394,3		3549	3154	2760	2366			,00427,3		3846	3418	2991	2564	
$\bar{2.73}$	,07581		0396	0791	1187	1582	1978	$\bar{2.98}$	,17858		0429	0857	1286	1715	2144
	,00395,5		3560	3164	2769	2373			,00428,7		3858	3430	3001	2572	
$\bar{2.74}$	,07976		0397	0794	1190	1587	1984	$\bar{2.99}$	,18286		0430	0860	1290	1720	2151
	,00396,8		3571	3174	2778	2381			,00430,1		3871	3441	3011	2581	

General Table III.  $\lambda(a^5)$ ,  $\lambda\left(\frac{1-a^5}{a-1}\right)$ .

$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a-1}\right)$	1 9	2 8	3 7	4 6	5	
1.00	18717	,00	0432	0863	1295	1726	2158	1.25	,29950	,00	0469	0939	1408	1877 2347
	,00431,6		3884	3453	3021	2590			,00469,3		4224	3754	3285	2816
1.01	19148		0433	0866	1299	1732	2165	1.26	,30419		0471	0942	1413	1884 2355
	,00433,0		3897	3464	3031	2598			,00470,9		4238	3767	3206	2825
1.02	19581		0434	0869	1304	1738	2175	1.27	,30890		0473	0945	1418	1890 2363
	,00434,5		3911	3476	3042	2607			,00472,5		4253	3780	3308	2835
1.03	20016		0435	0871	1306	1742	2177	1.28	,31363		0474	0948	1422	1896 2371
	,00435,4		3919	3483	3048	2612			,00474,1		4267	3793	3319	2845
1.04	20451		0438	0875	1313	1750	2188	1.29	,31837		0476	0951	1427	1903 2379
	,00437,6		3938	3401	3063	2626			,00475,7		4281	3806	3330	2854
1.05	20889		0439	0877	1316	1755	2194	1.30	,32312		0477	0955	1432	1909 2387
	,00438,7		3948	3510	3071	2632			,00477,3		4296	3818	3341	2864
1.06	21328		0440	0880	1321	1761	2201	1.31	,32790		0479	0958	1437	1916 2395
	,00440,2		3962	3522	3081	2641			,00479,0		4310	3831	3352	2873
1.07	21768		0442	0883	1325	1767	2209	1.32	,33269		0481	0961	1442	1922 2403
	,00441,7		3975	3534	3092	2650			,00480,6		4325	3845	3364	2884
1.08	22209		0443	0886	1330	1773	2216	1.33	,33749		0482	0964	1447	1929 2411
	,00443,2		3989	3546	3102	2659			,00482,2		4340	3858	3375	2893
1.09	22653		0445	0889	1334	1779	2224	1.34	,34232		0484	0968	1452	1936 2420
	,00444,7		4002	3558	3113	2668			,00483,9		4355	3871	3387	2903
1.10	23097		0446	0892	1338	1784	2231	1.35	,34715		0486	0971	1457	1942 2428
	,00446,1		4015	3569	3123	2677			,00485,5		4370	3884	3399	2913
1.11	23543		0448	0895	1343	1791	2240	1.36	,35201		0487	0974	1462	1949 2436
	,00447,7		4029	3582	3134	2686			,00487,2		4385	3898	3410	2923
1.12	23991		0449	0898	1348	1797	2246	1.37	,35688		0489	0978	1466	1955 2444
	,00449,2		4043	3594	3144	2695			,00488,8		4399	3910	3422	2933
1.13	24440		0451	0992	1353	1804	2255	1.38	,36177		0490	0981	1471	1962 2452
	,00450,9		4058	3607	3156	2705			,00490,4		4414	3923	3433	2942
1.14	24891		0452	0904	1356	1808	2260	1.39	,36676		0492	0984	1476	1968 2461
	,00452,0		4068	3616	3164	2712			,00492,1		4429	3937	3445	2953
1.15	25343		0454	0907	1361	1815	2269	1.40	,37159		0494	0988	1481	1975 2469
	,00453,7		4083	3630	3176	2722			,00493,8		4444	3950	3457	2903
1.16	25797		0455	091	1366	1821	2277	1.41	,37653		0496	0991	1487	1982 2477
	,00455,3		4098	3642	3181	2732			,00495,5		4460	3964	3469	2973
1.17	26252		0457	0914	1370	1827	2284	1.42	,38149		0497	0994	1492	1989 2486
	,00456,8		4111	3654	3198	2741			,00497,2		4475	3988	3480	2983
1.18	26709		0458	0917	1375	1833	2292	1.43	,38646		0499	0998	1497	1996 2495
	,00458,3		4125	3666	3208	2750			,00498,9		4490	3991	3492	2993
1.19	27167		0460	0920	1380	1840	2300	1.44	,39145		0501	1001	1502	2002 2503
	,00459,9		4139	3679	3219	2759			,00500,6		4505	4005	3504	3004
1.20	27627		0461	0923	1384	1846	2307	1.45	,39645		0502	1004	1507	2009 2511
	,00461,4		4153	3691	3230	2768			,00502,2		4520	4018	3515	3013
1.21	28089		0463	0926	1389	1852	2315	1.46	,40148		0504	1008	1512	2016 2520
	,00463,0		4167	3704	3241	2778			,00504,0		4536	4032	3528	3024
1.22	28551		0465	0929	1394	1858	2323	1.47	,40652		0506	1011	1517	2023 2529
	,00464,6		4181	3717	3252	2788			,00505,7		4551	4046	3540	3034
1.23	29016		0466	0932	1398	1864	2331	1.48	,41157		0507	1015	1522	2030 2537
	,00466,1		4194	3729	3263	2797			,00507,4		4567	4059	3552	3044
1.24	29482		0468	0935	1403	1871	2339	1.49	,41665		0509	1018	1527	2036 2546
	,00467,7		4209	3742	3274	2806			,00509,0		4582	4073	3564	3055

### General Table III. $\lambda(a^3), \lambda\left(\frac{1-a^3}{a^3-1}\right)$ .

General Table IV. For the whole of life. —  $\lambda(a-1)$ .

$\lambda(a)$	$-\lambda(a-1)$										
700	,00206	725	,05372	750	,10886	775	,16826	800	,23292	825	,30431
	,00201		,00213		,00229		,00247		,00271		,00302
701	,00407	726	,05585	751	,11115	776	,17073	801	,23564	826	,30733
	,00201		,00214		,00230		,00248		,00272		,00304
702	,00608	727	,05799	752	,11345	777	,17322	802	,23836	827	,31037
	,00202		,00215		,00230		,00249		,00274		,00305
703	,00810	728	,06014	753	,11575	778	,17571	803	,24110	828	,31342
	,00202		,00215		,00231		,00250		,00275		,00307
704	,01012	729	,06229	754	,11806	779	,17822	804	,24385	829	,31649
	,00203		,00216		,00232		,00251		,00270		,00308
705	,01214	730	,06445	755	,12037	780	,18073	805	,24661	830	,31957
	,00203		,00216		,00233		,00252		,00277		,00309
706	,01418	731	,06661	756	,12270	781	,18325	806	,24938	831	,32266
	,00203		,00217		,00233		,00253		,00278		,00311
707	,01621	732	,06878	757	,12503	782	,18578	807	,25216	832	,32577
	,00204		,00217		,00234		,00254		,00279		,00313
708	,01825	733	,07095	758	,12736	783	,18832	808	,25495	833	,32890
	,00205		,00219		,00235		,00254		,00281		,00314
709	,02030	734	,07314	759	,12971	784	,19086	809	,25776	834	,33204
	,00205		,00218		,00235		,00256		,00281		,00315
710	,02235	735	,07532	760	,13206	785	,19342	810	,26057	835	,33519
	,00205		,00219		,00236		,00257		,00283		,00319
711	,02440	736	,07751	761	,13442	786	,19599	811	,26340	836	,33836
	,00206		,00220		,00237		,00258		,00284		,00319
712	,02646	737	,07971	762	,13679	787	,19856	812	,26624	837	,34155
	,00207		,00221		,00237		,00259		,00285		,00320
713	,02853	738	,08192	763	,13916	788	,20115	813	,26909	838	,34475
	,00207		,00221		,00238		,00259		,00287		,00322
714	,03060	739	,08413	764	,14154	789	,20374	814	,27196	839	,34797
	,00207		,00221		,00239		,00260		,00287		,00324
715	,03267	740	,08634	765	,14393	790	,20634	815	,27483	840	,35121
	,00208		,00223		,00240		,00261		,00289		,00325
716	,03476	741	,08857	766	,14633	791	,20896	816	,27772	841	,35446
	,00208		,00223		,00240		,00262		,00291		,00327
717	,03684	742	,09080	767	,14873	792	,21158	817	,28063	842	,35773
	,00209		,00223		,00241		,00263		,00291		,00329
718	,03893	743	,09303	768	,15114	793	,21421	818	,28354	843	,36102
	,00210		,00224		,00242		,00264		,00293		,00331
719	,04103	744	,09527	769	,15356	794	,21685	819	,28647	844	,36433
	,00210		,00225		,00243		,00265		,00294		,00332
720	,04313	745	,09752	770	,15599	795	,21951	820	,28941	845	,36765
	,00211		,00226		,00244		,00266		,00295		,00334
721	,04524	746	,09978	771	,15843	796	,22217	821	,29236	846	,37099
	,00211		,00226		,00244		,00267		,00297		,00336
722	,04735	747	,10204	772	,16087	797	,22484	822	,29533	847	,37435
	,00212		,00227		,00245		,00268		,00298		,00338
723	,04947	748	,10431	773	,16333	798	,22753	823	,29831	848	,37773
	,00212		,00227		,00246		,00269		,00299		,00339
724	,05159	749	,10658	774	,16579	799	,23022	824	,30130	849	,38112
	,00213		,00228		,00247		,00270		,00301		,00342

General Table IV. For the whole of life. —  $\lambda(a-1)$ .

$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$
1.850	,38454 ,00343	1.875	,47688 ,00401	1.900	,58683 ,00488	1.925	,72468 ,00635	1.950	,91357 ,00929	1.975	,1,22728 ,01824
1.851	,38797 ,00345	1.876	,48088 ,00405	1.901	,59171 ,00493	1.926	,73103 ,00642	1.951	,92286 ,00946	1.976	,1,24552 ,01899
1.852	,39142 ,00348	1.877	,48493 ,00407	1.902	,59664 ,00497	1.927	,73745 ,00650	1.952	,93232 ,00966	1.977	,1,26451 ,01980
1.853	,39490 ,00349	1.878	,48900 ,00410	1.903	,60161 ,00502	1.928	,74395 ,00659	1.953	,94198 ,00984	1.978	,1,28431 ,02071
1.854	,39839 ,00351	1.879	,49310 ,00412	1.904	,60663 ,00507	1.929	,75054 ,00668	1.954	,95182 ,01006	1.979	,1,30502 ,02170
1.855	,40190 ,00353	1.880	,49722 ,00416	1.905	,61170 ,00511	1.930	,75722 ,00676	1.955	,96188 ,01027	1.980	,1,32672 ,02278
1.856	,40543 ,00355	1.881	,50138 ,00419	1.906	,61681 ,00516	1.931	,76398 ,00685	1.956	,97215 ,01049	1.981	,1,34950 ,02398
1.857	,40899 ,00357	1.882	,50557 ,00422	1.907	,62197 ,00521	1.932	,77083 ,00695	1.957	,98264 ,01073	1.982	,1,37348 ,02583
1.858	,41256 ,00360	1.883	,50979 ,00425	1.908	,62718 ,00527	1.933	,77778 ,00704	1.958	,99337 ,01097	1.983	,1,39881 ,02683
1.859	,41616 ,00362	1.884	,51404 ,00428	1.909	,63245 ,00531	1.934	,78482 ,00715	1.959	,1,00434 ,01123	1.984	,1,42564 ,02853
1.860	,41978 ,00364	1.885	,51832 ,00431	1.910	,63776 ,00537	1.935	,79197 ,00724	1.960	,1,01557 ,01150	1.985	,1,45417 ,03047
1.861	,42342 ,00366	1.886	,52263 ,00435	1.911	,64313 ,00543	1.936	,79921 ,00735	1.961	,1,02707 ,01179	1.986	,1,48464 ,03269
1.862	,42708 ,00368	1.887	,52698 ,00438	1.912	,64856 ,00548	1.937	,80656 ,00746	1.962	,1,03886 ,01209	1.987	,1,51733 ,03526
1.863	,44076 ,00371	1.888	,53136 ,00442	1.913	,65404 ,00554	1.938	,81402 ,00758	1.963	,1,05095 ,01241	1.988	,1,55259 ,03829
1.864	,443447 ,00373	1.889	,53578 ,00445	1.914	,65958 ,00559	1.939	,82160 ,00769	1.964	,1,06336 ,01274	1.989	,1,59088 ,04189
1.865	,443820 ,00376	1.890	,54023 ,00449	1.915	,66517 ,00566	1.940	,82929 ,00781	1.965	,1,07610 ,01309	1.990	,1,63277 ,04626
1.866	,44496 ,00378	1.891	,54472 ,00452	1.916	,67083 ,00572	1.941	,83710 ,00793	1.966	,1,08919 ,01348	1.991	,1,67903 ,05163
1.867	,44574 ,00380	1.892	,54924 ,00456	1.917	,67655 ,00578	1.942	,84503 ,00807	1.967	,1,10267 ,01387	1.992	,1,73069 ,05849
1.868	,44954 ,00383	1.893	,55380 ,00460	1.918	,68233 ,00584	1.943	,85310 ,00820	1.968	,1,11654 ,01429	1.993	,1,78918 ,06745
1.869	,45337 ,00385	1.894	,55840 ,00464	1.919	,68817 ,00591	1.944	,86130 ,00833	1.969	,1,13083 ,01475	1.994	,1,85663 ,07968
1.870	,45722 ,00388	1.895	,56304 ,00467	1.920	,69408 ,00598	1.945	,86963 ,00848	1.970	,1,14558 ,01523	1.995	,1,93631 ,09741
1.871	,46110 ,00390	1.896	,56771 ,00472	1.921	,70006 ,00695	1.946	,87811 ,00863	1.971	,1,16081 ,01574	1.996	,2,03372 ,12544
1.872	,46500 ,00393	1.897	,57243 ,00476	1.922	,70611 ,00611	1.947	,88674 ,00878	1.972	,1,17655 ,01530	1.997	,2,15916 ,17459
1.873	,46893 ,00396	1.898	,57719 ,00479	1.923	,71222 ,00620	1.948	,89552 ,00894	1.973	,1,19285 ,01690	1.998	,2,33575 ,30153
1.874	,47289 ,00399	1.899	,58198 ,00485	1.924	,71842 ,00626	1.949	,90446 ,00911	1.974	,1,20975 ,01753	1.999	,2,63728

TABLE V.—Logarithms of the accommodated chances of living 10 years, deduced from the value of an annuity for 10 years, at 5 per cent. from the actual tables of mortality, and considered equal to a geometrical series of ten terms, of which the common ratio is the same as the first term, and the tenth term the accommodated chance; and to find the accommodated chance for 5, 7 years, &c. without a table calculated for the purpose, it may be considered sufficient to multiply by .5, .7, &c. the accommodated ratio in this table when extreme accuracy be not required.

Age.	Carlisle.	Deparcieux.	Northampton.	Age.	Carlisle.	Deparcieux.	Northampton.
0	1,6892	—	—	52	1,9172	1,9006	1,8523
1	1,6763	—	1,7044	53	1,9098	1,8957	1,8471
2	1,8699	—	1,8356	54	1,9013	1,8901	1,8417
3	1,9159	1,9166	1,8790	55	1,8915	1,8853	1,8357
4	1,9401	1,9315	1,9081	56	1,8803	1,8799	1,8294
5	1,9586	1,9411	1,9220	57	1,8680	1,8732	1,8228
6	1,9686	1,9486	1,9369	58	1,8513	1,8673	1,8156
7	1,9737	1,9544	1,9476	59	1,8435	1,8601	1,8081
8	1,9764	1,9592	1,9550	60	1,8318	1,8511	1,7998
9	1,9773	1,9637	1,9586	61	1,8243	1,8398	1,7908
10	1,9768	1,9669	1,9592	62	1,8171	1,8264	1,7811
11	1,9754	1,9679	1,9582	63	1,8090	1,8120	1,7699
12	1,9742	1,9669	1,9566	64	1,7974	1,7946	1,7576
13	1,9729	1,9658	1,9546	65	1,7860	1,7735	1,7431
14	1,9716	1,9704	1,9521	66	1,7703	1,7510	1,7267
15	1,9704	1,9628	1,9490	67	1,7506	1,7270	1,7083
16	1,9698	1,9609	1,9455	68	1,7107	1,7017	1,6879
17	1,9694	1,9600	1,9419	69	1,7005	1,6754	1,6651
18	1,9693	1,9586	1,9388	70	1,6689	1,6480	1,6402
19	1,9690	1,9574	1,9358	71	1,6319	1,6167	1,6126
20	1,9685	1,9559	1,9337	72	1,5936	1,5841	1,5823
21	1,9679	1,9554	1,9321	73	1,5563	1,5500	1,5487
22	1,9670	1,9549	1,9311	74	1,5269	1,5119	1,5117
23	1,9659	1,9544	1,9298	75	1,4940	1,4711	1,4723
24	1,9644	1,9540	1,9289	76	1,4642	1,4218	1,4308
25	1,9628	1,9534	1,9277	77	1,4344	1,3684	1,3846
26	1,9573	1,9531	1,9264	78	1,4007	1,3134	1,3307
27	1,9591	1,9524	1,9257	79	1,3538	1,2497	1,2644
28	1,9570	1,9521	1,9238	80	1,3134	1,1876	1,1900
29	1,9556	1,9518	1,9226	81	1,2582	1,1214	1,1101
30	1,9552	1,9514	1,9211	82	1,2043	1,0609	1,0234
31	1,9548	1,9514	1,9196	83	1,1765	2,9688	2,9341
32	1,9540	1,9514	1,9180	84	1,0727	2,8536	2,8592
33	1,9528	1,9515	1,9164	85	2,9939	2,7199	2,7813
34	1,9513	1,9517	1,9146	86	2,9166	2,5736	2,7003
35	1,9485	1,9522	1,9126	87	2,8490	2,4254	2,6149
36	1,9477	1,9528	1,9104	88	2,8055	2,1943	2,5369
37	1,9452	1,9534	1,9083	89	2,7537	3,9129	2,4179
38	1,9437	1,9527	1,8057	90	2,6695	3,5265	2,2414
39	1,9406	1,9517	1,9031	91	2,6658	3,0266	2,9356
40	1,9383	1,9506	1,9001	92	2,7323	4,3694	3,5037
41	1,9372	1,9488	1,8973	93	2,8031	5,2971	4,7375
42	1,9365	1,9466	1,8943	94	2,8355	—	5,5769
43	1,9365	1,9438	1,8915	95	2,8107	—	7,0496
44	1,9366	1,9403	1,8882	96	2,8279	—	—
45	1,9367	1,9361	1,8848	97	2,7589	—	—
46	1,9366	1,9308	1,8810	98	2,6695	—	—
47	1,9358	1,9263	1,8767	99	2,5111	—	—
48	1,9351	1,9200	1,8740	100	2,1629	—	—
49	1,9328	1,9158	1,8631	101	3,5689	—	—
50	1,9292	1,9098	1,8621	102	4,3245	—	—
51	1,9233	1,9027	1,8571	103	6,0595	—	—

TABLE VI.—Accommodated annual ratio for an unlimited period for every age  $\alpha$ 

$$\lambda r = \lambda \frac{1}{1} \left[ \frac{1}{1} a \right] - \lambda \frac{1}{0} \left[ \frac{1}{1} a \right] + \lambda 1,05.$$

$\alpha$	$\lambda r$ Carlisle.	$\lambda r$ Deparcieux.	$\lambda r$ Northampton.	$\alpha$	$\lambda r$ Carlisle.	$\lambda r$ Deparcieux.	$\lambda r$ Northampton.
0	1,98665	—	—	52	1,98390	1,98216	1,97950
1	1,99121	—	1,98517	53	1,98305	1,98240	1,97878
2	1,99313	—	1,98997	54	1,98212	1,98156	1,97802
3	1,99458	1,99399	1,99151	55	1,98112	1,98073	1,97721
4	1,99528	1,99446	1,99244	56	1,98005	1,97982	1,97635
5	1,99577	1,99473	1,99284	57	1,97887	1,97882	1,97542
6	1,99599	1,99493	1,99324	58	1,97763	1,97780	1,97445
7	1,99606	1,99505	1,99346	59	1,97637	1,97668	1,97341
8	1,99606	1,99513	1,99357	60	1,97514	1,97545	1,97230
9	1,99600	1,99519	1,99354	61	1,97400	1,97408	1,97111
10	1,99529	1,99519	1,99341	62	1,97281	1,97254	1,96983
11	1,99576	1,99514	1,99323	63	1,97154	1,97093	1,96843
12	1,99563	1,99503	1,99304	64	1,97014	1,96912	1,96693
13	1,99549	1,99491	1,99284	65	1,96858	1,96707	1,96526
14	1,99535	1,99478	1,99262	66	1,96685	1,96489	1,96346
15	1,99522	1,99464	1,99240	67	1,96491	1,96250	1,96150
16	1,99509	1,99450	1,99213	68	1,96273	1,96009	1,95936
17	1,99487	1,99438	1,99190	69	1,96029	1,95750	1,95703
18	1,99484	1,99425	1,99167	70	1,95755	1,95480	1,95448
19	1,99471	1,99413	1,99145	71	1,95440	1,95181	1,95171
20	1,99455	1,99398	1,99124	72	1,95111	1,94870	1,94869
21	1,99442	1,99388	1,99106	73	1,94784	1,94542	1,94541
22	1,99425	1,99375	1,99088	74	1,94467	1,94180	1,94186
23	1,99408	1,99364	1,99070	75	1,94185	1,93790	1,93812
24	1,99390	1,99350	1,99051	76	1,93888	1,93330	1,93423
25	1,99370	1,99338	1,99030	77	1,93591	1,92833	1,92994
26	1,99349	1,99323	1,99009	78	1,93265	1,92314	1,92503
27	1,99328	1,99308	1,98988	79	1,92863	1,91715	1,91916
28	1,99306	1,99293	1,98965	80	1,92461	1,91124	1,91246
29	1,99290	1,99277	1,98943	81	1,91891	1,90493	1,90509
30	1,99265	1,99259	1,98917	82	1,91491	1,89856	1,89697
31	1,99245	1,99241	1,98892	83	1,90939	1,89069	1,88844
32	1,99224	1,99223	1,98865	84	1,90344	1,88805	1,88110
33	1,99201	1,99202	1,98837	85	1,89657	1,86782	1,87361
34	1,99176	1,99181	1,98808	86	1,88978	1,85416	1,86599
35	1,99149	1,99158	1,98777	87	1,88369	1,84011	1,85803
36	1,99110	1,99134	1,98745	88	1,87972	1,81788	1,85069
37	1,99090	1,99109	1,98811	89	1,87481	1,78941	1,85454
38	1,99058	1,99077	1,98675	90	1,86660	1,75223	1,82243
39	1,99025	1,99043	1,98637	91	1,86560	1,70265	1,79303
40	1,98991	1,99006	1,98597	92	1,87056	1,63694	1,74992
41	1,98958	1,98967	1,98556	93	1,87595	1,52971	1,67376
42	1,98924	1,98924	1,98513	94	1,87840	—	1,55753
43	1,98890	1,98878	1,98469	95	1,87967	—	1,30505
44	1,98853	1,98828	1,98423	96	1,77774	—	—
45	1,98814	1,98771	1,98375	97	1,77140	—	—
46	1,98771	1,98714	1,98323	98	1,76333	—	—
47	1,98725	1,98655	1,98270	99	1,84829	—	—
48	1,98673	1,98590	1,98209	100	1,81282	—	—
49	1,98612	1,98526	1,98148	101	1,75663	—	—
50	1,98546	1,98456	1,98083	102	1,65421	—	—
51	1,98471	1,98386	1,98017	103	1,40266	—	—

TABLE VII.—Logarithm of Carlisle chance of living 5 years at every age  $a$ .

$a$	$\lambda$ chance.								
0	1,83232	20	1,98469	40	1,96915	60	1,91826	80	1,66927
1	1,89709	21	1,98457	41	1,96836	61	1,91483	81	1,64194
2	1,92823	22	1,98439	42	1,96790	62	1,91180	82	1,61095
3	1,95354	23	1,98405	43	1,96780	63	1,90864	83	1,57100
4	1,96747	24	1,98333	44	1,96808	64	1,90492	84	1,53422
5	1,97792	25	1,98213	45	1,96857	65	1,90067	85	1,50393
6	1,98376	26	1,98091	46	1,96918	66	1,89586	86	1,45652
7	1,98703	27	1,97967	47	1,96941	67	1,88838	87	1,40377
8	1,98869	28	1,97863	48	1,96915	68	1,87746	88	1,36691
9	1,98930	29	1,97804	49	1,96818	69	1,86279	89	1,34438
10	1,98911	30	1,97789	50	1,96676	70	1,84362	90	1,32483
11	1,98836	31	1,97783	51	1,96477	71	1,82305	91	1,34054
12	1,98754	32	1,97767	52	1,96269	72	1,80220	92	1,38021
13	1,98670	33	1,97736	53	1,96017	73	1,78348	93	1,41373
14	1,98593	34	1,97687	54	1,95660	74	1,76877	94	1,43933
15	1,98528	35	1,97611	55	1,95155	75	1,75508	95	1,47712
16	1,98490	36	1,97490	56	1,94461	76	1,74231	96	1,48337
17	1,98479	37	1,97349	57	1,93711	77	1,72712	97	1,44370
18	1,98476	38	1,97194	58	1,92973	78	1,71062	98	1,33099
19	1,98472	39	1,97044	59	1,92343	79	1,68963		

Logarithm of the Carlisle chance of living 10 years at every age  $a$ .

0	1,81023	19	1,96805	38	1,93973	57	1,84891	76	1,38425
1	1,88086	20	1,96682	39	1,93851	58	1,83836	77	1,33807
2	1,91526	21	1,96548	40	1,93772	59	1,82835	78	1,28163
3	1,94223	22	1,96406	41	1,93754	60	1,81893	79	1,22385
4	1,95677	23	1,96268	42	1,93731	61	1,81070	80	1,17320
5	1,96702	24	1,96136	43	1,93694	62	1,80018	81	1,09846
6	1,97213	25	1,96002	44	1,93626	63	1,78610	82	1,01472
7	1,97457	26	1,95873	45	1,93533	64	1,76771	83	2,93791
8	1,97540	27	1,95734	46	1,93395	65	1,74430	84	2,87860
	1,97523	28	1,95598	47	1,93211	66	1,71891	85	2,82876
10	1,97438	29	1,95490	48	1,92932	67	1,69058	86	2,79706
11	1,97326	30	1,95400	49	1,92478	68	1,66094	87	2,78398
12	1,97233	31	1,95273	50	1,91830	69	1,63157	88	2,78064
13	1,97146	32	1,95116	51	1,90938	70	1,59870	89	2,78371
14	1,97065	33	1,94929	52	1,89980	71	1,56536	90	2,80195
15	1,96996	34	1,94730	53	1,88990	72	1,52932	91	2,82391
16	1,96947	35	1,94526	54	1,88003	73	1,49411	92	2,82391
17	1,96918	36	1,94326	55	1,86981	74	1,45840	93	2,74473
18	1,96881	37	1,94138	56	1,85944	75	1,42434		

TABLE VII.—*continued.*Logarithm of the Carlisle chance of living 15 years for every age  $a$ .

$a$	$\lambda$ chance.								
0	1,79934	18	1,94744	36	1,91244	54	1,78495	72	1,14027
1	1,86922	19	1,94609	37	1,91080	55	1,77048	73	1,06511
2	1,90280	20	1,94471	38	1,90888	56	1,75530	74	2,99263
3	1,92893	21	1,94331	39	1,90669	57	1,73729	75	2,92827
4	1,94270	22	1,94173	40	1,90448	58	1,71582	76	2,84078
5	1,95230	23	1,94004	41	1,90231	59	1,69114	77	2,74184
6	1,95702	24	1,93823	42	1,90000	60	1,66256	78	2,64853
7	1,95936	25	1,93613	43	1,89712	61	1,63375	79	2,56823
8	1,96015	26	1,93364	44	1,89284	62	1,60238	80	2,49803
9	1,95995	27	1,93082	45	1,88687	63	1,56958	81	2,43900
10	1,95907	28	1,92792	46	1,87856	64	1,53648	82	2,39494
11	1,95784	29	1,92534	47	1,86922	65	1,49937	83	2,35164
12	1,95672	30	1,92316	48	1,85905	66	1,46123	84	2,31794
13	1,95551	31	1,92108	49	1,84820	67	1,41770	85	2,30588
14	1,95397	32	1,91905	50	1,83656	68	1,37157	86	2,28043
15	1,95209	33	1,91709	51	1,82421	69	1,32120	87	2,22768
16	1,95038	34	1,91538	52	1,81160	70	1,26797	88	2,11163
17	1,94885	35	1,91383	53	1,79853	71	1,20730		

Logarithm of the Carlisle chance of living 20 years for every age  $a$ .

0	1,78462	17	1,92652	34	1,88356	51	1,72007	68	2,94257
1	1,85412	18	1,92479	35	1,88059	52	1,69998	69	2,85542
2	1,88759	19	1,92295	36	1,87721	53	1,67599	70	2,77190
3	1,91369	20	1,92082	37	1,87349	54	1,64744	71	2,66383
4	1,92742	21	1,91821	38	1,86906	55	1,61410	72	2,54404
5	1,93699	22	1,91521	39	1,86329	56	1,57835	73	2,43202
6	1,94160	23	1,91197	40	1,85602	57	1,53949	74	2,33701
7	1,94376	24	1,90866	41	1,84692	58	1,49930	75	2,25311
8	1,94421	25	1,90528	42	1,83711	59	1,45991	76	2,18132
9	1,94328	26	1,90199	43	1,82684	60	1,41763	77	2,12205
10	1,94120	27	1,89872	45	1,81628	61	1,37606	78	2,06227
11	1,93874	28	1,89572	45	1,80513	62	1,32950	79	2,00757
12	1,93639	29	1,89342	46	1,79339	63	1,28021	80	3,97515
13	1,93414	30	1,89172	45	1,78101	64	1,22611	81	3,92237
14	1,93201	31	1,89027	48	1,76768	65	1,16864	82	3,83863
15	1,92999	32	1,88847	49	1,75312	66	1,10317	83	3,68263
16	1,92821	33	1,88624	50	1,73723	67	1,02866		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 25 years at every age  $a$ .

$a$	$\lambda$ chance.								
0	1,76930	16	1,90311	32	1,85116	48	1,64514	64	2,76034
1	1,83969	17	1,90001	33	1,84641	49	1,61591	65	2,67257
2	1,87198	18	1,89673	34	1,84016	50	1,58086	66	2,55969
3	1,89774	19	1,89339	35	1,83213	51	1,5431	67	2,43242
4	1,91075	20	1,88997	36	1,82182	52	1,50218	68	2,30948
5	1,91912	21	1,88657	37	1,81060	53	1,45948	69	2,19980
6	1,92251	22	1,88311	38	1,79878	54	1,41651	70	2,09673
7	1,92342	23	1,87977	39	1,78672	55	1,36918	71	2,00437
8	1,92283	24	1,87674	40	1,77428	56	1,32067	72	3,92425
9	1,92131	25	1,87335	41	1,76175	57	1,26661	73	3,84575
10	1,91909	26	1,87117	42	1,74891	58	1,20993	74	3,77634
11	1,91657	27	1,86813	43	1,73548	59	1,14954	75	3,73023
12	1,91406	28	1,86487	44	1,72120	60	1,08690	76	3,66469
13	1,91150	29	1,86159	45	1,70581	61	1,01800	77	3,56575
14	1,90888	30	1,85848	46	1,68926	62	2,94045	78	3,39326
15	1,90610	31	1,85504	47	1,66940	63	2,85121		

Logarithm of the Carlisle chance of living 30 years at every age  $a$ .

$a$	$\lambda$ chance.								
0	1,75143	15	1,87525	30	1,81003	45	1,54943	60	2,59083
1	1,81960	16	1,87146	31	1,79964	46	1,51231	61	2,47452
2	1,85164	17	1,86790	32	1,78827	47	1,47160	62	2,34422
3	1,87637	18	1,86453	33	1,77614	48	1,42863	63	2,21811
4	1,88879	19	1,86147	34	1,76358	49	1,38467	64	2,10472
5	1,89701	20	1,85854	35	1,75039	50	1,33594	65	3,99740
6	1,90033	21	1,85575	36	1,73665	51	1,28544	66	3,90023
7	1,90109	22	1,85253	37	1,72240	52	1,22930	67	3,81264
8	1,90019	23	1,84892	38	1,70742	53	1,17010	68	3,72321
9	1,89818	24	1,84492	39	1,69164	54	1,10614	69	3,63913
10	1,89520	25	1,84061	40	1,67496	55	1,03845	70	3,57385
11	1,89147	26	1,83594	41	1,65761	56	2,96261	71	3,48774
12	1,88755	27	1,83083	42	1,63729	57	2,87756	72	3,36795
13	1,88344	28	1,82504	43	1,61294	58	2,78093	73	3,17674
14	1,87931	29	1,81819	44	1,58399	59	2,68376		

Logarithm of the Carlisle chance of living 35 years for every age  $a$ .

$a$	$\lambda$ chance.								
0	1,72933	14	1,84739	28	1,75476	42	1,43949	56	2,41913
1	1,79743	15	1,84382	29	1,74162	43	1,39642	57	2,28133
2	1,82932	16	1,84065	30	1,72829	44	1,35277	58	2,14784
3	1,85373	17	1,83732	31	1,71448	45	1,30451	59	2,02815
4	1,86565	18	1,83368	32	1,70007	46	1,25462	60	3,91566
5	1,87312	19	1,82964	33	1,68477	47	1,19872	61	3,81506
6	1,87523	20	1,82530	34	1,66850	48	1,13925	62	3,72443
7	1,87458	21	1,82052	35	1,65106	49	1,07432	63	3,63185
8	1,87213	22	1,81522	36	1,63251	50	1,00520	64	3,54405
9	1,86861	23	1,80909	37	1,61078	51	2,92738	65	3,47452
10	1,86435	24	1,80152	38	1,58488	52	2,84025	66	3,38360
11	1,85983	25	1,79216	39	1,55443	53	2,74110	67	3,25633
12	1,85544	26	1,78055	40	1,51858	54	2,64036	68	3,05420
13	1,85123	27	1,76794	41	1,48066	55	2,54237		

TABLE VII.—*continued.*Logarithm of the Carlisle chance of living 40 years for every age  $a$ .

$a$	$\lambda$ chance.								
0	1,70544	13	1,82038	26	1,69538	39	1,32320	52	2,24402
1	1,72233	14	1,81557	27	1,67973	40	1,27366	53	2,10801
2	1,80280	15	1,81057	28	1,66340	41	1,22298	54	3,98474
3	1,82567	16	1,80542	29	1,64654	42	1,16661	55	3,86721
4	1,83609	17	1,80001	30	1,62896	43	1,10705	56	3,75967
5	1,84227	18	1,79385	31	1,61034	44	1,04240	57	3,66154
6	1,84359	19	1,78624	32	1,58845	45	2,97377	58	3,56157
7	1,84247	20	1,77684	33	1,56223	46	2,89656	59	3,46748
8	1,83992	21	1,76513	34	1,53130	47	2,80967	60	3,39278
9	1,83669	22	1,75233	35	1,49469	48	2,71025	61	3,29843
10	1,83292	23	1,73882	36	1,45556	49	2,60854	62	3,16813
11	1,82901	24	1,72494	37	1,41298	50	2,50913	63	4,96284
12	1,82486	25	1,71042	38	1,36836	51	2,38390		

Logarithm of the Carlisle chance of living 45 years for every age  $a$ .

$a$	$\lambda$ chance.								
0	1,67459	12	1,78755	24	1,62987	36	1,19787	48	2,07716
1	1,74068	13	1,78055	25	1,61109	37	1,14010	49	3,95292
2	1,77070	14	1,77217	26	1,59125	38	1,07899	50	3,83396
3	1,79346	15	1,76212	27	1,56812	39	1,01283	51	3,72444
4	1,80417	16	1,75002	28	1,54086	40	2,94292	52	3,62423
5	1,81084	17	1,73712	29	1,50933	41	2,86492	53	3,52174
6	1,81277	18	1,72357	30	1,47258	42	2,77756	54	3,42408
7	1,81190	19	1,70967	31	1,43339	43	2,67805	55	3,34433
8	1,80907	20	1,69510	32	1,39065	44	2,57662	56	3,24304
9	1,80487	21	1,67996	33	1,34571	45	2,47770	57	3,10524
10	1,79968	22	1,66412	34	1,30007	46	2,35308	58	4,89256
11	1,79378	23	1,64745	35	1,24977	47	2,21344		

Logarithm of the Carlisle chance of living 50 years for every age  $a$ .

$a$	$\lambda$ chance.								
0	1,64316	11	1,73839	22	1,55251	33	1,05634	44	3,92100
1	1,70987	12	1,72466	23	1,52491	34	2,98970	45	3,80253
2	1,74011	13	1,71228	24	1,49266	35	2,91903	46	3,69362
3	1,76261	14	1,69560	25	1,45471	36	2,83982	47	3,59365
4	1,77234	15	1,68038	26	1,41430	37	2,75105	48	3,49089
5	1,77760	16	1,66486	27	1,37032	38	2,65000	49	3,39225
6	1,77754	17	1,64892	28	1,32434	39	2,54705	50	3,31109
7	1,77458	18	1,63222	29	1,27810	40	2,44685	51	3,20781
8	1,76925	19	1,61459	30	1,22766	41	2,32144	52	3,06793
9	1,76147	20	1,59577	31	1,17570	42	2,18133	53	4,85274
10	1,75123	21	1,57582	32	1,11777	43	2,04495		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 55 years for every age  $a$ .

$a$	$\lambda$ chance.								
0	1,60991	10	1,66949	20	1,43940	30	2,89693	40	3,77168
1	1,67464	11	1,65322	21	1,39887	31	2,81764	41	3,66198
2	1,70281	12	1,63646	22	1,35471	32	2,72872	42	3,56155
3	1,72278	13	1,61892	23	1,30840	33	2,62734	43	3,45869
4	1,72894	14	1,60051	24	1,26143	34	2,52392	44	3,36033
5	1,72914	15	1,58105	25	1,20979	35	2,42296	45	3,27966
6	1,72215	16	1,56072	26	1,15661	36	2,29634	46	3,17699
7	1,71169	17	1,53730	27	1,09743	37	2,15482	47	3,03735
8	1,69897	18	1,50967	28	1,03497	38	2,01689	48	4,82189
9	1,68490	19	1,47738	29	2,96773	39	3,89144		

Logarithm of the Carlisle chance of living 60 years for every age  $a$ .

0	1,56146	9	1,58982	18	1,29315	27	2,70839	36	3,63689
1	1,61925	10	1,57016	19	1,24615	28	2,60597	37	3,53503
2	1,63992	11	1,54908	20	1,19448	29	2,50196	38	3,43063
3	1,65251	12	1,52484	21	1,14119	30	2,40086	39	3,33077
4	1,65237	13	1,49638	22	1,08183	31	2,27417	40	3,24881
5	1,64740	14	1,46331	23	1,01902	32	2,13249	41	3,14535
6	1,63698	15	1,42467	24	2,95106	33	3,99425	42	3,00524
7	1,62349	16	1,38377	25	2,87906	34	3,86830	43	4,78968
8	1,60761	17	1,33950	26	2,79855	35	3,74779		

Logarithm of the Carlisle chance of living 65 years for every age  $a$ .

0	1,47972	8	1,48507	16	1,12608	24	2,48528	32	3,51270
1	1,53408	9	1,45261	17	1,06662	25	2,38298	33	3,40798
2	1,55171	10	1,41378	18	1,00378	26	2,25507	34	3,30763
3	1,56115	11	1,37213	19	2,93578	27	2,11216	35	3,22492
4	1,55729	12	1,32704	20	2,86374	28	3,97288	36	3,12025
5	1,54807	13	1,27986	21	2,78313	29	3,84634	37	4,97873
6	1,53285	14	1,23208	22	2,69278	30	3,72569	38	4,76162
7	1,51187	15	1,17975	23	2,59002	31	3,61471		

Logarithm of the Carlisle chance of living 70 years for every age  $a$ .

0	1,38039	7	1,31497	14	2,92171	21	2,23965	28	3,38661
1	1,42994	8	1,26855	15	2,84902	22	2,09655	29	3,28567
2	1,44010	9	1,22138	16	2,76802	23	3,95693	30	3,20281
3	1,43861	10	1,16886	17	2,67757	24	3,82967	31	3,09808
4	1,42003	11	1,11445	18	2,57478	25	3,70782	32	4,95640
5	1,39170	12	1,05416	19	2,47001	26	3,59561	33	4,73897
6	1,35590	13	2,99049	20	2,36767	27	3,49237		

TABLE VII.—*continued.*Logarithm of the Carlisle chance of living 75 years for every age  $a$ .

$a$	$\lambda$ chance.								
0	1,22402	6	1,09821	12	2,66511	18	3,94169	24	3,26900
1	1,25299	7	1,04119	13	2,56149	19	3,81439	25	3,18494
2	1,24230	8	2,97918	14	2,45593	20	3,69250	26	3,07898
3	1,22209	9	2,91101	15	2,35295	21	3,58019	27	4,93607
4	1,18885	10	2,83813	16	2,22455	22	3,47676	28	4,71760
5	1,14678	11	2,75639	17	2,08134	23	3,37066		

Logarithm of the Carlisle chance of living 80 years for every age  $a$ .

0	2,97909	5	2,81604	10	2,34206	15	3,67778	20	3,16963
1	2,99530	6	2,74015	11	2,21291	16	3,56508	21	3,06356
2	2,69411	7	2,65214	12	2,06888	17	3,46155	22	4,92046
3	2,93272	8	2,55018	13	3,92839	18	3,35542	23	4,70166
4	2,87848	9	2,44523	14	3,80031	19	3,25372		

Logarithm of the Carlisle chance of living 85 years for every age  $a$ .

0	2,64836	4	2,41271	8	3,91708	12	3,44909	16	3,04845
1	2,63724	5	2,31997	9	3,78962	13	3,34213	17	4,90525
2	2,58037	6	2,19667	10	3,66689	14	3,23965	18	4,68641
3	2,50372	7	2,05591	11	3,55345	15	3,15490		

Logarithm of the Carlisle chance of living 90 years for every age  $a$ .

0	2,15229	3	3,87062	6	3,53721	9	3,22895	12	4,89279
1	2,09377	4	3,75709	7	3,43612	10	3,14401	13	4,67312
2	3,98414	5	3,64480	8	3,33082	11	3,03682		

Logarithm of the Carlisle chance of living 95 years for every age  $a$ .

0	3,47712	2	3,36435	4	3,19642	6	3,02058	8	4,66181
1	3,43431	3	3,28436	5	3,12193	7	4,87982		

Logarithm of the Carlisle chance of living 100 years for every age  $a$ .

0	4,95424	1	4,91768	2	4,80805	3	4,61535
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TABLE VIII.—Logarithm of Deparcieux chance of living for every age  $a$ .

$a$	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0									
1									
2									
3	1,93450	1,89763	1,85126	1,80346	1,73957	1,62634	1,39967	2,85126	3,30103
4	1,94469	1,90644	1,85957	1,81188	1,74401	1,62495	1,37684	2,78408	3,01323
5	1,95159	1,91193	1,86455	1,81698	1,74418	1,61979	1,34747	2,70443	
6	1,95683	1,91575	1,86784	1,82040	1,74248	1,61130	1,31482	2,61130	
7	1,96027	1,91825	1,86981	1,82177	1,73928	1,59968	1,27663	2,50098	
8	1,96282	1,91985	1,87151	1,82222	1,73410	1,58512	1,23231	2,38721	
9	1,96495	1,92101	1,87278	1,82146	1,72821	1,56781	1,18415	2,25473	
10	1,96614	1,92212	1,87309	1,81970	1,72110	1,54688	1,12740	2,09691	
11	1,96582	1,92042	1,87239	1,81612	1,71269	1,52337	1,06380	3,90458	
12	1,96448	1,91860	1,87069	1,81067	1,70296	1,49545	2,99190	3,60454	
13	1,96313	1,91676	1,86896	1,80507	1,69184	1,46517	2,91676	3,36653	
14	1,96175	1,91488	1,86719	1,79932	1,68026	1,43215	2,83939	3,06854	
15	1,96034	1,91296	1,86539	1,79259	1,66820	1,39588	2,75284		
16	1,95892	1,91101	1,86357	1,78565	1,65447	1,35799	2,65447		
17	1,95798	1,90954	1,86150	1,77901	1,63941	1,31636	2,54071		
18	1,95703	1,90869	1,85940	1,77128	1,62230	1,26949	2,42439		
19	1,95606	1,90783	1,85651	1,76326	1,60286	1,21920	2,28978		
20	1,95508	1,90695	1,85356	1,75496	1,58074	1,16126	2,13077		
21	1,95460	1,90657	1,85030	1,74687	1,55755	1,09798	2,93876		
22	1,95412	1,90621	1,84619	1,73848	1,53097	1,02742	2,70006		
23	1,95363	1,90583	1,84194	1,72871	1,50204	2,95363	3,40340		
24	1,95313	1,90544	1,83757	1,71851	1,47040	2,87764	3,10679		
25	1,95262	1,90505	1,83225	1,70786	1,43554	2,79250			
26	1,95209	1,90465	1,82673	1,69555	1,39907	2,69555			
27	1,95156	1,90352	1,82103	1,68143	1,35838	2,58273			
28	1,95166	1,90237	1,81425	1,66527	1,31246	2,46736			
29	1,95177	1,90045	1,80720	1,64180	1,26314	2,33372			
30	1,95187	1,89848	1,79988	1,62568	1,20618	2,17569			
31	1,95197	1,89570	1,79227	1,60295	1,14338	2,98416			
32	1,95209	1,89207	1,78436	1,57685	1,07330	2,74594			
33	1,95220	1,88831	1,77508	1,54841	1,00000	2,44977			
34	1,95231	1,88444	1,76538	1,51727	2,92451	3,15366			
35	1,95243	1,87963	1,75524	1,48292	2,83988				
36	1,95256	1,87464	1,74346	1,44698	2,74346				
37	1,95196	1,86947	1,72987	1,40682	2,63117				
38	1,95071	1,86259	1,71361	1,36080	2,51570				
39	1,94868	1,85543	1,69503	1,31137	2,38195				
40	1,94661	1,84801	1,67379	1,25431	2,22382				
41	1,94373	1,84030	1,65098	1,19141	2,03219				
42	1,93998	1,83227	1,62476	1,12121	2,79385				
43	1,93611	1,82288	1,59621	1,04780	2,49757				
44	1,93213	1,81307	1,56496	2,97220	2,20135				
45	1,92720	1,80281	1,53049	2,88745					
46	1,92208	1,79090	1,49442	2,79090					
47	1,91751	1,77791	1,45486	2,67921					
48	1,91188	1,76290	1,41009	2,56499					
49	1,90675	1,74635	1,36269	2,43327					
50	1,90140	1,72718	1,30770	2,27721					
51	1,89657	1,70725	1,24768	2,08846					
52	1,89229	1,68478	1,18123	2,85387					

[continued.]

TABLE VIII. *continued.*—Logarithm of Deparcieux chance of living for every age  $a$ .

TABLE IX.—Logarithm of the Northampton chance for living at every age  $a$ .

$a$	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0	1,68764	1,64396	1,57564	1,49417	1,38958	1,24287	1,02428	2,60484	3,59643
1	1,81295	1,76713	1,69746	1,61431	1,50640	1,35435	1,12443	2,67151	3,59446
2	1,88378	1,83536	1,76454	1,67952	1,56899	1,41046	1,16788	2,67677	3,51790
3	1,91089	1,85979	1,78780	1,70070	1,58568	1,42229	1,16522	2,62961	3,37283
4	1,92894	1,87511	1,80190	1,71263	1,59383	1,42421	1,15070	2,55993	3,14495
5	1,93843	1,88180	1,80733	1,71581	1,59300	1,41691	1,12431	2,47370	4,80625
6	1,94739	1,88788	1,81211	1,71823	1,59118	1,40806	1,09339	2,37854	
7	1,95322	1,89101	1,81390	1,71755	1,58601	1,39522	1,05661	2,27263	
8	1,95660	1,89203	1,81352	1,71459	1,57827	1,37909	1,01505	2,15453	
9	1,95739	1,89080	1,81084	1,70923	1,56781	1,35940	2,96901	2,03386	
10	1,95632	1,88800	1,80653	1,70194	1,55523	1,33664	2,91720	3,90879	
11	1,95418	1,88451	1,80136	1,69345	1,54140	1,31148	2,85856	3,78151	
12	1,95158	1,88076	1,79574	1,68431	1,52668	1,28410	2,79299	3,63412	
13	1,94890	1,87691	1,78981	1,67479	1,51140	1,25433	2,71872	3,46194	
14	1,94617	1,87296	1,78369	1,66489	1,49527	1,22176	2,63099	3,21602	
15	1,94337	1,86890	1,77738	1,65457	1,47848	1,18588	2,53527	4,86782	
16	1,94049	1,86472	1,77084	1,64379	1,46667	1,14600	2,43115		
17	1,93779	1,86068	1,76433	1,63279	1,44200	1,10339	2,31941		
18	1,93543	1,85692	1,75799	1,62167	1,42249	1,05845	2,19793		
19	1,93341	1,85345	1,75184	1,61042	1,40201	1,01162	2,07647		
20	1,93168	1,85021	1,74562	1,59891	1,38032	2,96088	3,95247		
21	1,93033	1,84718	1,73927	1,58722	1,35730	2,90438	3,82733		
22	1,92918	1,84417	1,73274	1,57511	1,33253	2,84141	3,68255		
23	1,92801	1,84091	1,72589	1,56250	1,30543	2,76982	3,51304		
24	1,92679	1,83752	1,71872	1,54910	1,27559	2,68482	3,26984		
25	1,92553	1,83401	1,71120	1,53511	1,24251	2,59191	4,92445		
26	1,92423	1,83035	1,70330	1,52018	1,20551	2,49066			
27	1,92289	1,82654	1,69500	1,50421	1,16560	2,38162			
28	1,92149	1,82256	1,68624	1,48706	1,12302	2,26250			
29	1,92004	1,81843	1,67701	1,46860	1,07821	2,14306			
30	1,91853	1,81394	1,66723	1,44864	1,02920	2,02079			
31	1,91685	1,80894	1,65689	1,42697	2,07405	3,89700			
32	1,91498	1,80355	1,64592	1,40334	2,91223	3,75336			
33	1,91290	1,79788	1,63449	1,37742	2,84181	3,58503			
34	1,91073	1,79193	1,62231	1,34880	2,75802	3,34305			
35	1,90848	1,78567	1,60958	1,31698	2,66637	4,99892			
36	1,90612	1,77907	1,59595	1,28128	2,56643				
37	1,90365	1,77211	1,58132	1,24271	2,45873				
38	1,90107	1,76475	1,56557	1,20153	2,34101				
39	1,89839	1,75697	1,54856	1,15817	2,22302				
40	1,89541	1,74870	1,53011	1,11067	2,10226				
41	1,89209	1,74004	1,51012	1,05720	2,98015				
42	1,88857	1,73094	1,48836	2,99725	2,83838				
43	1,88498	1,72159	1,46452	2,92891	2,67213				
44	1,88120	1,71158	1,4307	2,84730	2,43232				
45	1,87719	1,70110	1,40850	2,7579	3,09044				
46	1,87295	1,68983	1,37516	2,66031					
47	1,86846	1,67767	1,33906	2,55508					
48	1,86368	1,66450	1,30046	2,43994					
49	1,85858	1,65017	1,25978	2,32463					
50	1,85329	1,63470	1,21526	2,20685					
51	1,84795	1,61803	1,16511	2,08806					
52	1,84237	1,59979	1,10868	2,94981					

[continued.]

TABLE IX. *continued.*—Logarithm of the Northampton chance for living at every age  $\alpha$ .

How the value of particular assurances may be determined from the value of annuities, is shown in my Paper in the Philosophical Transactions for the year 1820, many of the cases of which are solved by methods essentially the same as those which have been long adopted; but when such assurances are but for terms, which are not of great extension, very near approximations may be had by using a geometrical progression, without confining the arithmetical operations to the same route, since the chance of extinction of the joint lives of the present age  $a, b, c, \&c.$  taking place between the period commencing with the time  $n+t-1$ , and finishing with the time  $n+t$ , from the present, is  $= \frac{L_{n+t-1}: a, b, c, \&c. - L_{n+t}: a, b, c, \&c.}{L_{a, b, c, \&c.}}$ ; it follows that if  $r$  be the present value of unity, to be received certain in the time 1, and  $L_{n+t-1}: a, b, c, \&c. = L_{n-1}: a, b, c, \times r^t$ , whatever  $t$

may be, that  $\frac{r}{m} \left[ a, b, c, \&c. \right]$  or the assurance of unity to be received at the first of the equal periods 1, from the commencement of the time  $n-1$  to the expiration of the time  $m$ , which shall happen after the extinction of the joint lives, is equal to  $\frac{L_{n-1}: a, b, c, \&c.}{L_{a, b, c, \&c.}} \times \{ r^n \times (1-\pi) + r^{n+1} \times (\pi - \pi^2) + r^{n+2} \times (\pi^2 - \pi^3) \dots r^m \times (\pi^{m-n-1} - \pi^{m-n}) \} = (1-\pi) \times \frac{L_{n-1}: a, b, c, \&c.}{L_{a, b, c, \&c.}} \times \{ r + \pi r + \pi^2 r + \pi^3 r + \dots r^m \pi^{m-n-1} \} = \frac{(1-\pi) r}{L_{a, b, c, \&c.}} \times \{ r^{n-1} L_{n-1}: a, b, c, \&c. + r \cdot L_{n-1}: a, b, c, \&c. + \dots + r \cdot L_{m-1}: a, b, c, \&c. \} = (1-\pi) \cdot r \times \frac{r}{m-1} \left[ a, b, c, \&c. \right]$

If the assurance be not deferred,  $n$  will be equal to 1, and

we shall have, according to the hypothesis,  $\frac{r}{m} \left[ a, b, c, \&c. \right] = (1-\pi) \cdot r \times \frac{r}{m} \left[ a, b, c, \&c. \right]$ ; and also  $= \frac{1-\pi}{\pi} \cdot \frac{1}{m} \left[ a, b, c, \&c. \right]$ . If  $t$  be

taken equal to 1, we shall have from the equation  $L_{n+t-1:a, b, c, \&c.} =$

$L_{n-1:a, b, c, \&c.} \times \pi^t$ ,  $\pi = \frac{L_{n:a, b, c, \&c.}}{L_{n-1:a, b, c, \&c.}}$ , and this would be the real value which should be taken for  $\pi$ , if the geometrical progression coincided perfectly with the fact; and it would be

indifferent whether we made it equal to  $\frac{L_{n+t:a, b, c, \&c.}}{L_{n-1+t:a, b, c, \&c.}}$ , or  $\frac{L_{n:a, b, c, \&c.}}{L_{n-1:a, b, c, \&c.}}$ , as the two would be the same; but this not

being the case, there will be a preference; and generally, if not always,  $\pi$  should be taken an intermediate value between the two; and when the term is not very long, it will answer a good purpose to take it about the middle between them, inclining generally, though perhaps not always, rather nearer the last than the first, as the first terms are generally of more consequence than the last. If the said assurance be not deferred, and instead of being paid for immediately, be to be paid for by equal periodic payments, at an unite of time from each other, up to the time  $m-1$  inclusive, and the first payment be to be made immediately, then will the

present value of such periodic payment be  $\frac{r}{m-1} \underline{a, b, c, \&c.}$ , and consequently each payment, from what is shown above, is

equal to  $\frac{r}{m} \underline{a, b, c, \&c.} \div \frac{r}{m-1} \underline{a, b, c, \&c.} = (1-\pi). r$ . From whence we may draw an inference worthy of remark, namely; when an assurance of joint lives is meant to commence immediately, and to continue for a term of  $t$  years, which is not large, and to be paid for by  $t$  annual payments, that those payments will not differ much with the increase of the time  $t$ , provided, as I have said, that  $t$  be not large, and the ages

be not at the extremes of life, a consequence which follows from the near agreement to a geometrical progression which takes place in the number of living at each small equal increment of time; that is to say, from the near coincidence

of  $\frac{L}{n : a, b, c, \&c.}$  with  $\frac{L}{n-1 : a, b, c, \&c.}$  with  $\frac{L+t : a, b, c, \&c.}{n-1 : a, b, c, \&c.}$ , or the small variation of  $\pi$  for the different values of  $t$ : and also, that when the number of years for which an assurance continues be not very long, and the ages be not at the extremes of life, the annual premiums will not differ widely from the premiums to be paid for an assurance of one year of a life older than the proposed life by about half the term: thus, according to the Northampton table, at three per cent. to assure 100*l.* at the

Age . . . . .	15	20	30	40	50	60	64
For 7 years, the annual premium by the common modes of calculation	£1..2..11	1..9..5	1..14..11	2..4..1	3..0..8	4..7..1	5..4..10
And the premium for one year assurance for an age 3 years older . . . . .	1..3..3	1..9..8	1..15..0	2..4..6	3..1..0	4..7..8	5..5..6

the difference of which is very small.—As another example, let

Age . . . . .	10	20	30	40	50	60
For 10 years, the annual premium will be, by common modes of calculation	£0..19..2	1..9..1	1..15..8	2..5..8	3..3..4	4..12..6
Premium for one year assurance, age 5 years older	0..17..11	1..10..7	1..16..4	2..6..8	3..5..1	4..15..2

Here, except at the age 10, the excess is rather more in the approximation than in the first set of examples; but it should be recollected, that we took the exact middle, instead of inclining to the early age.

According to the Carlisle table of mortality at 3 per cent.  
to assure 100*l.* at the

Age . . . .	10	20	30	40	50	60
For 7 years, the annual premium, by common modes of calculation .	£0 10 5	0 13 10	0 19 10	1 7 8	1 11 0	3 13 8
For one year, the premium	0 10 5	0 13 9	0 19 2	1 8 6	1 12 1	3 15 9
For 10 years, the annual premium, by common modes of calculation .	0 11 3	0 14 7	1 0 4	1 7 7	1 14 11	3 17 8
For one year, at an age 5 years older . . . .	0 12 0	0 14 2	0 19 11	1 9 0	1 14 10	3 19 9

Moreover, because  $\underline{\substack{1 \\ a, b, c, \&c.}}$ , or the single premium for the assurance of unity, on the joint lives  $a, b, c, \&c.$  for  $m$

years, is  $= \underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}} \cdot r - \underline{\substack{1 \\ m \\ a, b, c, \&c.}} = \underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}} \cdot r + 1 -$

$\frac{\underline{\substack{1 \\ m \\ a, b, c, \&c.}} \cdot r^m}{\underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}} = 1 - \frac{\underline{\substack{1 \\ m \\ a, b, c, \&c.}} \cdot r^m}{\underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}} \cdot (1-r) \cdot \underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}};$

if this be divided by  $\underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}$ , we shall have the annual

premium for such assurance; that is,  $\frac{\underline{\substack{1 \\ m \\ a, b, c, \&c.}}}{\underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}} - 1 + r$ . The said annual premium may be expressed by

$$\left(1 - \frac{\underline{\substack{1 \\ m \\ a, b, c, \&c.}} \cdot r^m}{\underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}} \right) \div \left( \frac{\underline{\substack{1 \\ m \\ a-1, b-1, c-1, \&c.}} \cdot \frac{\underline{\substack{1 \\ m-1 \\ a-1, b-1, c-1, \&c.}}}{\underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}} \right) - 1 + r.$$

This last mode is well adapted to logarithms in the use of our general tables; and this method, supposing the annuities were accurately determinable by our general tables, would be accurate. The last formula is derived from that imme-

diately before, in consequence of  $\underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}$  being identical

with  $\underline{\substack{1 \\ m \\ a-1, b-1, c-1, \&c.}} \times \frac{\underline{\substack{1 \\ m-1 \\ a-1, b-1, c-1, \&c.}}}{r \underline{\substack{1 \\ m-1 \\ a, b, c, \&c.}}}$

Example. To find the annual premium to assure a life, at the age  $a$  years, for 10 years, according to the Carlisle mortality, and three per cent. interest.

$a =$	20	30	40	50	60	70
Log. of the accommoda. chance for living 10 yrs. at the age $a-1$ , Tab. V.	.9690	.9556	.9406	.9328	.8435	.7005
$\lambda_{10}^{a-1} =$	.8716	.8716	.8716	.8716	.8716	.8716
Sum . . . =	.8406	.8272	.8122	.8044	.7151	.5721
Corresponding . . . To this we get from Ta. I.	.91443 31	.90407 362	.89892 103	.89379 205	.84846 248	.78092 94
$\lambda_{10}^{\frac{1}{a-1}} . . . =$	.91474	.90779	.90005	.89605	.85099	.78191
Therefore, $\lambda_{10}^{\frac{a+10}{L}} \text{ (T. VII.)}$	.96682	.95400	.93772	.91830	.81893	.59873
$\lambda_{10}^{a-10} =$	.87163	.87163	.87163	.87163	.87163	.87163
Sum = the log. . . .	.83845	.82563	.80935	.78993	.69056	.47036
The N° corresponding =	.68937	.66932	.64469	.61650	.49041	.29536
Its complement to unity	.31063	.33068	.35531	.38350	.50959	.70464
The log. of the last . =	.49224	.51941	.55061	.58377	.70722	.84797
Complement of $\lambda_{10}^{\frac{1}{a-1}}$ =	.08526	.09221	.09995	.10395	.14901	.21809
$\lambda_{10}^{\frac{a}{L-1}} . . . . . =$	.99694	.99571	.99481	.99402	.98754	.97813
$\lambda_{10}^{r-1} . . . . . =$	.98716	.98716	.98716	.98716	.98716	.98716
Sum = logarithm . . .	.56160	.59449	.63253	.66890	.83093	.03135
Number corresponding =	.03644	.03931	.04291	.04666	.06775	.10749
$\lambda_{10}^{a-1} =$	.02913	.02913	.02913	.02913	.02913	.02913
Ann. premium for an assurance of 1l. . . . =	.00732	.01018	.01378	.01753	.03862	.07836
Ditto for 100l. . . . =	£0..14..8	1.. 0.. 4	1.. 7.. 7	1..15.. 1	3..1 .. 3	7..16.. 9

} for annual  
premiums.

The reader has here an opportunity of comparing the results from my tables, with those above calculated by Mr. MILNE's Carlisle tables.— I may probably be able at a future period to add examples, which I regret time will not at present permit.